No. of Printed Pages : 3 **MMT-005**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCES)

M. Sc. (MACS)

Term-End Examination

December, 2022

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : (*i*) *Question No.* ***1*** *is compulsory.*

(*ii*) *Attempt any* ***three*** *questions from Question Nos.* ***2*** *to* ***5****.*

(*iii*) *Use of calculators is not allowed.*

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1. State, giving reasons, whether the following statements are True *or* False : 5×2=10

   (i) If *f* is analytic and real-valued on a domain D, then *f* is constant.

P. T. O.

(ii) A multiply connected domain is not connected.

(iii) If a function f is analytic at all points interior to and on a simple contour r, $\int_r f = 0$.

(iv) The radius of convergence of the power series $\sum_{n=3}^{\infty} \frac{2^n}{n^2}(z-2-i)^n$ is $\frac{1}{2}$.

(v) $z = 0$ is an isolated essential singularity for $f(z) = z^2 \sin \frac{1}{z}$.

2. (a) Find the Möbius transformation mapping -1 to 0, ∞ to 1 and i to ∞. 3

(b) Find the velocity vector, velocity potential and stream function for the flow whose complex potential is $\Phi(z) = z^3$. 2

3. (a) Show that $u(x, y) = xy + 3x^2y - y^3$ is harmonic in the entire complex plane $\mathbb{C}$. Also find the harmonic conjugate function of u. Is the harmonic conjugate function of u unique ? Why ? 3

(b) Sketch the image of the vertical strip $0 < \operatorname{Re} z < 1$ under the exponential map $w = e^z$. 2

4. (a) Find the Laurent expansion centred at 0 of the function *f*, defined by $f(z) = \dfrac{2}{z(z^2 - 1)}$ in the annular region $0 < |z| < 1$. 2

(b) Consider an analytic function *f*, defined in a domain D. Let $\alpha \in D$ be such that $f'(\alpha) \neq 0$. Prove that *f* is conformal at α. 3

5. (a) Evaluate the integral : 3

$$\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}.$$

(b) Find the maximum modulus of $f(z) = e^z + z + 1$ on $|z| \leq 1$. 2