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M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

December, 2022

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : *Question No. **5** is **compulsory**. Answer any **three** questions from Q. Nos. **1** to **4**. Use of calculators is **not** allowed.*

1. Let T be a linear operator on $\mathbf{R}^3$ whose matrix with respect to an ordered basis $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$ is $\begin{bmatrix}2 & 1 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$. Check whether or not T is a bijection. If it is, find the matrix of

T^{-1} with respect to the ordered basis $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$. If T is not bijective, check whether T is normal. 5

2. Find the singular value decomposition of the matrix $\begin{bmatrix}2 & 1\\1 & 1\\-1 & 3\end{bmatrix}$. Hence obtain the Moore-Penrose inverse of the given matrix. 5

3. (a) Write the Jordan form of the matrix $\begin{bmatrix}2 & 1 & 1\\0 & 2 & 2\\0 & 0 & 2\end{bmatrix}$. Is this matrix similar to $\begin{bmatrix}2 & 0 & 1\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$? Give reasons for your answer. 3

(b) Find the QR decomposition of $\begin{bmatrix}3\\1\\0\end{bmatrix}$. 2

4. (a) Let M and A be a metro city and a nearby small town, respectively. Each year 20% of A's population moves to M and 5% of M's population moves to A. What is the long-term effect of this on the populations of M and A ? Are they likely to stabilise ? Why or why not ? 3

(b) Find a unitary matrix whose first column is $\frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix}$. Further, check whether or not this unitary matrix is unitarily diagonalisable. 2

5. Which of the following statements are true ? Give reasons for your answers : 10

(i) The sum of two diagonalisable matrices is a diagonalisable matrix.

(ii) The only diagonalisable nilpotent matrix is the zero matrix.

(iii) There is no unitary matrix having one of its entries 2.

(iv) If A is an $n \times n$ matrix such that det (A) > 0, then A is a positive definite matrix.

(v) If $A \in \mathbf{M}_n(\mathbf{C})$ has an eigen value with algebraic multiplicity greater than one, then A cannot be normal.