No. of Printed Pages : 4 **MTE-09**

BACHELOR'S DEGREE PROGRAMME

(BDP)

Term-End Examination

December, 2022

MTE-09 : REAL ANALYSIS

Time : 2 Hours *Maximum Marks : 50*

Note : *Attempt **five** questions in all. Question No. **1** is compulsory. Attempt any **four** questions from Question Nos. **2** to **7**. Use of calculator is not allowed.*

1. State whether the following statements are true *or* false ? Give reasons for your answers : 2 each

   (i) The set] –3, 4 [U] – 4, 3[is a neighbourhood of 2.

   (ii) Every alternating series is convergent.

   (iii) If a continuous function $f : [2, 3] \to \mathbf{R}$ changes sign in [2, 3], then $f(x) \neq 0$, for all x in [2, 3].

P. T. O.

(iv) The function f defined on $\mathbf{R}$ by $f(x) = -|x-2|$ has a local maxima at $x = 2$.

(v) If sum of two real functions f and g is integrable, then both f and g are also integrable.

2. (a) Show that the function :

$$f(x) = |3-x| + 2x - x^2$$

is continuous at $x = 3$ but not differentiable thereat. 4

(b) Prove that between any two real roots of the equation $\cos x = e^{-x}$, there is at least one root of $e^x \sin x = 1$. 3

(c) Prove that a lower bound 'ℓ' of a non-empty subset S of $\mathbf{R}$ is the infimum of S only if for every $\varepsilon > 0$, there exists an $s_\varepsilon \in S$ such that $s_\varepsilon \in \ell + \varepsilon$. 3

3. (a) Show that $\{0, 2, 8\} \cup \{1, 5, 5^2, 5^4, 5^8\}$ is a closed set. 2

(b) Let $\langle a_n \rangle$ be a sequence defined as $a_1 = 1$ and $a_{n+1} = \frac{1}{4} a_n$ for all n. Check whether or not $\langle a_n \rangle$ is convergent. If convergent, find its limit. 3

(c) Prove that the sequence of functions $(f_n(x))$, where $f_n(x) = \frac{nx}{1 + n^2x^2}$ is pointwise convergent but not uniformly convergent in [–1, 1]. 5

4. (a) Define an algebraic number. Show that $\sqrt{3} - 2$ is an algebraic number. 2

(b) Prove that for $x > 0$, $\sqrt[3]{1 + x} < 1 + \frac{1}{3}x$. 4

(c) Test the integrability of the function f depend on $[a, b]$ by : 4

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ 3, & \text{when } x \text{ is irrational} \end{cases}$$

5. (a) Justify that : 2

$$\lim_{x \to -2} \frac{1}{(x + 2)^2} = \infty$$

(b) Test the following series for convergence : 4

(i) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$

(ii) $\sum_{n=1}^{\infty} \frac{3n - 7}{n^2 + 5n}$

(c) Find $\lim_{n \to \infty} \sum_{r=1}^{2n} \frac{1}{4n - 3r}$. 4

6. (a) Using Weierstrass M-test, show that the series $\sum_{n=1}^{\infty}(n^2-1)x^n$ converges uniformly in $\left[-\frac{1}{2},\frac{1}{2}\right]$. 3

(b) Examine the function $(x-1)^5(3x+1)^3$ for extreme at the point $x=\frac{1}{6}$. 4

(c) Prove that the function $f(x)=x^{-1}$ is not uniformly continuous on]0, 1]. 3

7. (a) For any two real numbers a and b, prove that : 3

$$|a-b| \geq \big||a|-|b|\big|.$$

(b) On the curve, $y=2x^2-x+1$, prove that the chord joining the points whose abscissa are $x=2$ and $x=3$ is parallel to the tangent to the curve at the point, whose abscissa is $x=2.5$. 3

(c) Test the absolute and the conditional convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{3n+5}$. 4