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BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (REVISED)

Term-End Examination

December, 2021

BCS-012 : BASIC MATHEMATICS

Time : 3 Hours *Maximum Marks : 100*

Note : *Question number **1** is compulsory. Attempt any **three** questions from the remaining questions.*

1. (a) Find the inverse of matrix : 5

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

(b) If 7 times the 7th term of an A.P. is equal to 11 times the 11th term of the A.P., find its 18th term. 5

(c) If z is a complex number such that $|z - 2i| = |z + 2i|$, show that Im $(z) = 0$. 5

(d) Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$. 5

(e) Use the principle of mathematical induction to show that : 5

$$1 + 4 + 7 + \ldots\ldots + (3k - 2) = \frac{1}{2}k(3k - 1)$$

(f) Evaluate $\int \frac{dx}{e^x + 1}$. 5

(g) Find the quadratic equation whose roots are $(2 - \sqrt{3})$ and $(2 + \sqrt{3})$. 5

(h) Find the length of the curve :

$$y = 3 + \frac{1}{2}(x)$$

from (0, 3) to (2, 4). 5

2. (a) Find the shortest distance between : 5

$$\vec{r_1} = (1+\lambda)\,\hat{i} + (2-\lambda)\,\hat{j} + (1+\lambda)\,\hat{k}$$

and $\vec{r_2} = 2\,(1+\mu)\,\hat{i} + (1-\mu)\,\hat{j} + (-1+2\mu)\,\hat{k}$

(b) Find the points of local minima and local maxima, for function : 5

$$f(x) = \frac{3}{4}x^4 - 8x^3 + \frac{45}{2}x^2 + 2015$$

(c) Find the sum of all integers between 100 and 1000 which are divisible by 7. 5

(d) If $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 5 & 2 \\ 2 & -1 & 7 \end{bmatrix}$, show that A is row equivalent to I_3. 5

3. (a) A stone is thrown into a lake, producing circular ripple. The radius of the ripple is increasing at the rate of 5 m/s. How fast is

the area inside the ripple increasing when the radius is 10 m ? 5

(b) If $(x + iy)^{1/3} = a + ib$, prove that : 5

$$\frac{x}{a} + \frac{y}{b} = 4\,(a^2 - b^2)$$

(c) Find the 10th term of the harmonic progression : 5

$$\frac{1}{7}, \frac{1}{15}, \frac{1}{23}, \frac{1}{31}, \ldots\ldots\ldots$$

(d) For any two vectors $\vec{a}$ and $\vec{b}$, show that : 5

$$\left|\vec{a} + \vec{b}\right| \leq \left|\vec{a}\right| + \left|\vec{b}\right|$$

4. (a) Determine the values of x for which :

$$f(x) = 5x^{3/2} - 3x^{5/2},\; x > 0$$

is increasing and decreasing. 5

(b) Solve the following system of liner equations by using matrix inverse : 10

$$3x + 4y + 7z = -2$$

$$2x - y + 3z = 6$$

$$2x + 2y - 3z = 0$$

and hance, obtain the value of $3x - 2y + z$.

(c) Find the area bounded by the curves $y = x^2$ and $y^2 = x$. 5

5. (a) If $y = \left(x + \sqrt{x^2 + 1}\right)^3$, find $\frac{dy}{dx}$. 5

(b) A company wishes to invest at most \$ 12,000 in project A and project B. Company must invest at least \$ 2,000 in project A and at least \$ 4,000 in project B.

If project A gives return of 8% and project B gives return of 10%, find how much money is to be invested in the two projects to maximize the return. 10

(c) Solve the equation :

$$2x^3 - 15x^2 + 37x - 30 = 0$$

if roots of the equation are in A. P. 5