M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

February, 2021

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any **five** questions. Use of calculator is **not** allowed.*

1. (a) Find the number of quantities required for estimating the expected return and standard deviation for 250 securities in Markowitz model. How many estimates are required for the securities while using single-index Sharpe model ? *2*

(b) The return distribution of two securities X and Y are given in the table below :

Possible Rates of Return		Associated Probabilities
X	Y	$P_{Xj} = P_{Yj}$
0·10	0·09	0·20
0·11	0·11	0·22
0·17	0·16	0·25
0·19	0·18	0·33

Find σ_{XY} and P_{XY}. *3*

2. Consider the discrete time population model given by $N_{t+1} = \dfrac{rN_t}{1+\left(\dfrac{N_t}{K}\right)^b}$ for a population, where r is the intrinsic growth rated, b is a positive parameter. Determine the non-negative steady-state and discuss the linear stability of the model for $0 < r < 1$. Also find the first bifurcation value of the parameters. *5*

3. (a) Let the returns on three securities A, B and C be 30%, 25% and 15%, respectively with $\sigma_A = 5$, $\sigma_B = 6$, $\sigma_C = 7$, $\sigma_{AB} = \sigma_{AC} = 16$ and $\sigma_{BC} = -10$. Find the standard deviation σ_P of the portfolio P = (0·4, 0·1, 0·5). *2*

(b) Let G(t) be the amount of the glucose in the bloodstream of a patient at time t. The glucose is infused into the bloodstream at a constant rate of R gm/min. At the same time, the glucose is converted and removed from the bloodstream at a rate proportional to the amount of glucose present. If the initial concentration of glucose in the bloodstream was G_0, then find the concentration at any time t. Also find the limiting value of the concentration. *3*

4. (a) A company has three factories F_1, F_2, F_3 and these factories supply to three markets M_1, M_2, M_3. The transportation costs from each factory to each market are given in the table. Capacities 'a_i's' of the factories and market requirements 'b_j's' are also shown in the table. Find the minimum transportation cost. *3*

	M_1	M_2	M_3	a_i
F_1	2	1	3	20
F_2	1	2	3	30
F_3	2	1	2	10
b_j	10	10	20	40/60

(b) A simple model including the seasonal change that affects the growth rate of a population is given by $\frac{dx}{dt} = Cx(t) \cos t$ where C is a constant. If x_0 is the initial population, solve the equation and determine the maximum and minimum population. *2*

5. (a) The yearly fluctuations in the groundwater table is believed to be dependent on the annual rainfall. The data collected on these variables for five consecutive years is given below :

Water table (in cm)	Annual rainfall (in cm)
1	1
2	2
3	2
4	3
5	3

Use the method of least squares to find the regression equation of the form $y = ax^2 + b$ that best fits the data. *3*

(b) Indifference curves of an investor cannot intersect. Is this true ? Give reasons for your answer. *2*

6. Ships arrive at a port at the rate of one in every 4 hours with exponential distribution of inter arrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 10 hours. If the average delay of ships waiting for a berth is to be kept below 14 hours, how many berths should be provided at the port ? *5*