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MMT-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

00032

Term-End Examination

December, 2018

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Question no. 1 is **compulsory**. Attempt any **three** questions from questions no. **2** to **5**. Use of calculators are **not** allowed.*

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1. State, giving reasons, whether the following statements are *True* or *False* : *5×2=10*

   (a) The function $f(z) = \bar{z}$ is nowhere differentiable.

   (b) The series $\sum_{n=0}^{\infty} \frac{1}{n!} z^n$ has radius of convergence zero.

(c) For any simple closed contour C such that $0 \not\subset C$

$$\int_C \frac{1}{z}\, dz \neq 0.$$

(d) If $a = e^{i\theta}$, then a^i represents infinitely many real numbers.

(e) Inverse mapping of a Mobius transformation is a Mobius transformation.

2. (a) Show that the function

$$f(z) = f(x, y) = \frac{xy(x + iy)}{x^2 + y^2}, \quad z \neq 0$$

$$= 0, \quad z = 0$$

is not differentiable at $z = 0$. *3*

(b) Show that the series $\sum_{n=1}^{\infty} \frac{z^{n+1}}{n}$ converges at all points inside the circle $|z| = 1$. What can you say about the convergence on the circle $|z| = 1$? *2*

3. (a) Consider $f(z) = z^2 - 2z + 4$ and the closed circular region $R = \{z : |z| \leq 2\}$. Find points in R where $|f(z)|$ has its maximum and minimum values. *3*

(b) Evaluate $\int_C \frac{dz}{z^2}$ where the contour C is the ellipse $(x-2)^2 + \frac{1}{4}(y-5)^2 = 1$. 2

4. (a) Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for $1 < |z-2| < 2$. 3

(b) Find a linear fractional transformation that maps the points 1, i and -1 on the unit circle $|z| = 1$ onto the points $-1, 0, 1$ on the real axis. Determine the image of the interior of $|z| < 1$ under this transformation. 2

5. Using contour integration, evaluate 5

$$\int_{-\infty}^{\infty} \frac{\sin x}{x(x^2 - 2x + 2)}\, dx.$$