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RST-003

Ph.D. IN STATISTICS (PHDSTAT)

Term-End Examination

00211 **December, 2017**

RST-003 : ADVANCED SAMPLE SURVEYS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***four*** *questions from questions no. 2 to 7. Only non-programmable scientific calculator is allowed. Symbols have their usual meanings.*

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1. (a) State whether the following statements are *true* or *false*. Give reasons in support of your answers. *8*

(i) Randomly selecting schools, and then sampling everyone within the school, is an example of simple random sampling.

(ii) The total number of all possible samples of size 3 from a population of size 5 is 10.

(iii) If the estimated value of MSE is 50 and its actual value is 60, then the relative error is 0·20.

(iv) In systematic sampling, the sample mean $\bar{y}_{sys}$ is an unbiased estimator of the population mean.

(b) Differentiate between the following, with suitable examples to illustrate the differences : *12*

(i) Simple random sampling with replacement and Probability proportional to size sampling

(ii) Sampling and Non-sampling errors

(iii) Linear and Circular systematic sampling

(iv) Two-stage and Two-phase sampling

2. A population is divided into two strata of sizes N_1 and N_2 and standard deviations S_1 and S_2. The cost of the survey is fixed and given by $C = c_1n_1 + c_2n_2$. Assuming that S_1 and S_2 are nearly equal and finite population correction can be ignored,

(a) Show that

$$\frac{V_{prop}}{V_{opt}} = \frac{N(N_1C_1 + N_2C_2)}{(N_1\sqrt{C_1} + N_2\sqrt{C_2})^2}, \text{ and}$$

(b) Compute the relative increase in precision using proportional allocation when $\frac{C_2}{C_1} = 2$ and $N_1 = N_2$. *20*

3. (a) A random sample of size 10, each being a one-acre plot (sampling unit = one-acre plot), from a 1000-acre plantation was taken using simple random sampling without replacement. The investigator counted the number of trees (y) on each selected plot and estimated the number of trees (x) from the aerial photographs of the plots. The data is given as follows :

Plot No.	Actual Number per Acre (y)	Estimated Number from Aerial Photographs (x)
1	25	23
2	15	14
3	22	20
4	24	25
5	13	12
6	18	18
7	35	30
8	30	27
9	10	8
10	29	31

Estimate the total number of trees in the 1000-acre plantation using the method of regression estimation. Also compute its relative efficiency as compared to the estimate based on simple random sampling. It is given that population mean based on aerial photographs $(\overline{X}) = 19{\cdot}7$.

(b) Describe the ratio method of estimation under double sampling scheme, and give a suitable example. *20*

4. A restaurant chain wishes to estimate the job satisfaction per employee (the scale is from 1 to 7). There are 120 restaurants and the total number of employees in the chain is 6860. A sample of 4 restaurants was taken using Simple Random Sampling Without Replacement (SRSWOR). Then again an SRSWOR sample of employees is taken from each selected restaurant and selected employees are interviewed. The data is given as follows :

Restaurant	M_i	Employee Satisfaction
1	52	5, 7, 6, 5, 4, 7, 6, 4, 5, 4
2	45	7, 7, 7, 6, 5, 4, 7, 7, 6
3	38	7, 6, 7, 6, 6, 5
4	65	7, 6, 5, 4, 6, 5, 7, 4, 6, 5, 6, 7

where M_i denotes the total number of employees in the i^{th} restaurant, $\forall$ i = 1, 2, 3, 4. Estimate the average employee job satisfaction along with its standard error, using any two estimators under this scheme. *20*

5. (a) If $\bar{y}_{st}$, $\bar{y}_{sy}$, and $\bar{y}$ are the estimators of population mean under stratified, systematic and simple random sampling, then prove that

$$V(\bar{y}_{st}) : V(\bar{y}_{sy}) : V(\bar{y}) \cong 1 : n : n^2,$$

assuming the population has a linear trend.

(b) Derive the expression for relative efficiency of cluster sampling as compared to SRSWOR in terms of the intra-class correlation coefficient. *12+8*

6. Differentiate between separate and combined ratio estimators. Also derive and compare their variances. *20*

7. The yields (in kg) of 8 orchards with 50, 30, 25, 40, 26, 44, 20 and 35 trees are 60, 35, 30, 44, 30, 50, 22 and 40, respectively. Estimate the total production of the 8 orchards along with the standard error of estimate using the

(a) Des Raj Ordered Estimator,

(b) Murthy's Unordered Estimator,

(c) Horvitz-Thompson Estimator.

For this purpose, consider the orchards placed 5^{th} and 7^{th} as selected in a sample of size 2. *20*
