No. of Printed Pages : 7 **MST-003**

POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

00362

Term-End Examination

December, 2017

MST-003 : PROBABILITY THEORY

Time : 3 hours *Maximum Marks : 50*

Note :

*(i) Attempt **all** questions. Questions no. 2 to 5 have internal choices.*

(ii) Use of scientific calculator is allowed.

(iii) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.

(iv) Symbols have their usual meanings.

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1. State whether the following statements are *True* or *False*. Give reasons in support of your answers. *5×2=10*

   (a) One of the two events is certain to happen. The chance of one event is one-fifth of the other, then odds in favour of the other event are 1 : 5.

(b) The mean and variance of a binomial variate $X \sim B\left(6, \frac{2}{3}\right)$ are found to be 4 and $\frac{4}{3}$.

(c) If $X \sim N(7, 12)$ and $Y \sim N(4, 9)$ are independent normal variates, then $(X - Y) \sim N(11, 3)$.

(d) The probability of drawing two heart cards from a pack of cards without replacement is $\frac{1}{26}$.

(e) If A and B are two independent events with $P(A) = 0{\cdot}3$, $P(B) = 0{\cdot}4$, then $P(A \cup B) = 0{\cdot}55$.

2. (a) The numbers on a roulette wheel are 0, 1, 2, ..., 36 and 00. The numbers 0 and 00 are green, while the other even numbers are red and the odd numbers are black. Assuming that the wheel is fair, find the probability that the outcome is (i) 13 or green, (ii) greater than 17 and black, (iii) red or black. *4*

(b) A die is thrown as long as necessary for face one to turn up. Assuming that face one does not turn out at the first throw,

(i) what is the probability that more than 3 throws will be necessary ?

(ii) Suppose that the number n of throws are required to get the first one as even. What is the probability for n = 2 ? *6*

OR

(a) Give a suitable example to describe the Law of Multiplication Probability. *3*

(b) Suppose that in a college 1 male student out of 5 and 1 female student out of 20 have science honours. A science honours student is chosen at random from a population that has twice as many males as females. What is the probability that the student is (i) male, (ii) female ? *7*

3. (a) A random variable X takes values 0, 1, 2, ... with probability proportional to $(x + 1) \left(\frac{1}{5}\right)^x$. Find the probability that $X \leq 5$. *4*

(b) A biased coin is tossed three times, with $P(H) = \frac{2}{3}$ and $P(T) = \frac{1}{3}$. Let the random variable X represent the number of heads produced in three tosses of the said coin. Find the CDF of X. *3*

(c) Describe the probability density function and probability mass function with suitable example of each. *3*

OR

(a) The distribution function F of a continuous variable is given by

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ x^2, & \text{if } 0 \le x \le \frac{1}{2} \\ 1 - \frac{3(3-x)^2}{25}, & \text{if } \frac{1}{2} \le x < 3 \\ 1, & \text{if } x \ge 3 \end{cases}$$

Find the pdf of X and evaluate $P(|X| \le 1)$ and $P\left(\frac{1}{3} \le X < 4\right)$, using both F and f. *6*

(b) Calculate the E(X) of the following functions : *4*

(i) $f(x) = \frac{1}{2\sqrt{x}}, 0 < x < 1$

(ii) $f(x) = \frac{1}{2} x^2 e^{-x}, 0 < x < \infty$

4. (a) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1·8. Find the probability that this computer will function for a month (i) without a breakdown, (ii) with only one breakdown, and (iii) with at least one breakdown. *5*

(b) A taxi cab company has 12 Ambassadors and 8 Fiats. If 5 of these taxi cabs are in the workshop for repairs and an Ambassador is as likely to be in for repairs as a Fiat, what is the probability that (i) 3 of them are Ambassadors and 2 are Fiats, (ii) at least 3 of them are Ambassadors, and (iii) all 5 are of the same make ? *5*

OR

(a) Find the probability that a person tossing 3 fair coins gets either all heads or all tails for the second time on the fifth trial. *3*

(b) If a body is throwing stones at a target, what is the probability that his 10^{th} throw is his 5^{th} hit, if the probability of hitting the target at any trial is $\frac{1}{2}$? *3*

(c) A manufacturer, who produces bolts, finds that 0·1% of the bolts are defective. The bolts are packed in boxes containing 500 bolts. A service centre buys 100 boxes from the producer. Find the expected number of boxes which will contain at least two defective bolts ? *4*

5. (a) In a certain city, the daily consumption of electric power in millions of kilowatt-hours can be treated as a random variable having a Gamma distribution with parameters $\lambda = \frac{1}{2}$ and $r = 3$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day ? *5*

(b) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least 1 of them would have a score above 75 ? *5*

OR

(a) The marks obtained by the students in Statistics in an examination are normally distributed with mean 150 and standard deviation 14·14. Find the probability that a student selected at random has secured a total of (i) 180 or above, and (ii) 135 or less. 5

(b) The mileage which car owners get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000 km. Find the probability that one of these tyres will last (i) at least 20,000 km, and (ii) at most 30,000 km. 5