No. of Printed Pages : 4

MMTE-001

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00191

Term-End Examination

December, 2017

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*
(Weightage : 50%)

Note : *Question no. 1 is* ***compulsory*** *and carries 10 marks. Answer any* ***four*** *out of questions no. 2 to 7. Computational devices such as electronic calculators, watches, etc. are* ***not*** *allowed.*

---

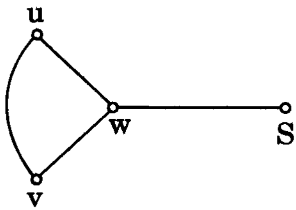
1. (a) Define Isomorphism between graphs. Are the graphs G_1 and G_2 isomorphic ? Explain your answer. *4*

(b) Define the k-dimensional cube Q_k. Is Q_k bipartite ? Justify your answer. *3*

(c) Let G be a connected graph and V be its vertex set. Show that the function $d : V \times V \to \mathbf{Z}$, defined by d(u, v), which represents the number of edges of the shortest u-v path, is a metric on V. *3*

2. (a) Define chromatic number $\chi(G)$ and clique number $\omega(G)$ of a graph G. State and prove a relation between them. *4*

(b) Find all maximal paths, maximal cliques and maximal independent sets in the graph. *3*



(c) Suppose G is a connected, bipartite graph. Prove that G has a unique bipartition. *3*

3. (a) If G and H are two simple graphs with vertex set V, then prove that $d_G(v) = d_H(v)$ for every $v \in V$ if, and only if, there is a sequence of 2-switches that transforms G into H. *6*

(b) Let u and v be adjacent vertices in a graph G with n vertices. Prove that uv belongs to at least $d(u) + d(v) - n$ triangles in G. *4*

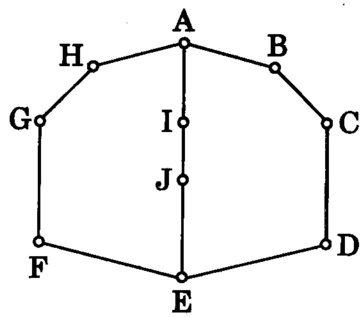
4. (a) Prove that Kruskal's algorithm constructs a minimum-weight spanning tree in a connected weighted graph G. *6*

(b) Show that the graph $K_{2,3}$ is planar and $K_{3,3}$ is not planar. *4*

5. (a) If a matching M in a graph G has no M-augmenting path, then show that M is maximum. Is the converse true ? Justify your answer by either giving a counter example or by giving a proof of the converse statement. 4

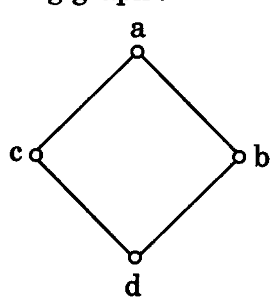
(b) If G is a connected graph which is neither a complete graph nor an odd cycle, then show that $\Delta(G) \geq \chi(G)$. 6

6. (a) Check whether the following graph is Hamiltonian. Justify your answer. 3



(b) Show that every Eulerian bipartite graph has an even number of edges. 3

(c) Use Mycielski's construction to construct a 3-chromatic, triangle-free graph from the following graph : 4



7. State, with justifications or illustrations, whether each of the following statements is *True* or *False* : *5×2=10*

(a) Every tree with two or more vertices is bipartite.

(b) For every $k \in \mathbf{N}$, every k-regular bipartite graph has a perfect matching.

(c) If u and v are the only vertices of odd degree in a graph G, then G contains a u-v path.

(d) The sequence (4, 4, 4, 3, 3, 3, 2, 2, 2, 1, 1, 1) is a graphic sequence.

(e) If P is a $u - v$ path in a 2-connected graph G, then there is a $u - v$ path Q internally disjoint from P.