No. of Printed Pages : 4

MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

00221

December, 2017

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

***Note :** Attempt any **five** questions. Use of scientific non-programmable calculator is allowed.*

1. (a) What is a Mathematical Model ? Discuss the terms : (i) Continuous Model (ii) Discrete Model (iii) Stochastic Model, giving example of each. *2*

(b) From the table

x	2	9	3	5	1
y	1	17	3	9	0

find a best fit line to estimate the value of y for x = 6. *3*

2. Find the solution of the finite population model

$$P_n - P_{n-1} = B_n P_{n-1} - D_n P_{n-1} + M_n,$$

where B_n and D_n are per capita birth and death rates respectively and M_n is migration in n^{th} generation. The initial population P_0 is constant. Discuss the case when $B_n = B$, $D_n = D$, $M_n = M$ (all fixed) and $B < D$ or $B > D$. 5

3. Consider a discrete model given by

$$N_{t+1} = \frac{rN_t}{1 + bN_{t-1}^2} = f(N_t),\ 1 < r < \frac{8}{7}.$$

Investigate the linear stability about the non-zero positive steady state N^* by setting $N_t = N^* + n_t$. Show that n_t satisfies the difference equation $n_{t+1} - n_t + 2(r-1)^{-1} rn_{t-1} = 0$. 5

4. A bank has two tellers working on the savings accounts. The first teller only handles withdrawals. The second teller only handles deposits. It has been found that the service time distribution for the deposits and withdrawals, both, are exponential with mean service time 3 minutes per customer. Depositors are found to arrive in a Poisson fashion throughout the day with a mean arrival rate of 16 per hour. Withdrawers also arrive in a Poisson fashion with a mean arrival rate of 14 per hour. What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both the withdrawals and deposits ? What would be the effect, if this could only be accomplished by increasing the service time to 3·5 minutes ? 5

5. Consider the advection-diffusion-reaction model of a single species population given by the following equation :

$$\frac{\partial N}{\partial t} = rN\left(1 - \frac{N-1}{K}\right) + D\frac{\partial^2 N}{\partial x^2} - v\frac{\partial N}{\partial x}, 0 \leq x \leq 2$$

The initial and boundary conditions are as follows :

$$N(x, 0) = f(x) > 0 \ \forall \ x; \ 0 \leq x \leq 2$$

$$N = N_i^* \ \text{at} \ x = 0 \ \text{and} \ x = 2 \ \forall \ t, \ i = 1, 2$$

where N_i^* are the steady-state solutions of the given model.

Obtain the steady-state solutions and do the stability analysis of non-zero steady state solution. *5*

6. (a) Let the return distribution of two securities 1 and 2 be as given below :

Possible rate of return		Probabilities
1	2	
0·18	0·11	$\frac{1}{4}$
0·13	0·08	$\frac{1}{2}$
0·06	0·07	$\frac{1}{4}$

Find the expected return of the portfolio $P = (w_1, w_2)$ when 70% of the total funds are invested in security 1 and remaining 30% in security 2. *$2\frac{1}{2}$*

(b) In a tumour region, the control parameters of growth and decay of a tumour are 65 and 35 per month respectively. Emigration occurs at a constant rate of 3×10^3 cells per month. Use these assumptions to formulate the logistic model of the tumour size. Solve the formulated equation and describe the long term behaviour of the tumour size when the initial size of the tumour is 5×10^6 cells. $2\frac{1}{2}$