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BICE-027

B.Tech. – VIEP – MECHANICAL ENGINEERING / B.Tech. CIVIL ENGINEERING (BTMEVI / BTCLEVI)

Term-End Examination

00812

December, 2017

BICE-027 : MATHEMATICS-III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any* ***ten*** *questions. All questions carry equal marks. Use of scientific calculator is permitted.*

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1. Expand for $f(x) = k$ for $0 < x < 2$ in a half range sine series. 7

2. Find the Fourier series expansion of the periodic function of period 2π

$$f(x) = x^2, \; -\pi \leq x \leq \pi.$$

Hence, find the sum of the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$$ 5+2=7

3. Find the Fourier series for the function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 1 - x, & 1 < x < 2 \end{cases}.$$ 7

4. Solve : 7

$$(x^2 - y^2 - z^2)\, p + 2xy\, q = 2xz$$

5. Solve : 7

$$\frac{\partial^3 z}{\partial x^3} - \frac{\partial^3 z}{\partial y^3} = x^3 y^3$$

6. Solve : 7

$$(D - 3D' - 2)^2\, z = 2e^{2x} \sin (y + 3x)$$

7. Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u, \text{ where } u(x, 0) = 6e^{-3x}.$$ 7

8. Solve completely the equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2},$$

representing the vibrations of a string of length l, fixed at both ends, given that

$$y(0, t) = 0,\ y(l, t) = 0,\ y(x, 0) = f(x) \text{ and}$$

$$\frac{\partial}{\partial t} y(x, 0) = 0,\ 0 < x < l.$$ 7

9. Find the temperature in a bar of length 2 whose ends are kept at zero and lateral surface insulated if the initial temperature is

$$\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}.$$ 7

10. Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < \pi,\ 0 < y < \pi,$$

which satisfies the conditions

$u(0, y) = u(\pi, y) = u(x, \pi) = 0$ and

$u(x, 0) = \sin^2 x.$ 7

11. A thin rectangular plate whose surface is impervious to heat flow has t = 0 an arbitrary distribution of temperature f(x, y). Its four edges x = 0, x = a, y = 0, y = b are kept at zero temperature. Determine the temperature at a point of the plate as t increases. 7

12. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 + (x/a), & -a < x < 0 \\ 1 - (x/a), & 0 < x < a \\ 0, & \text{otherwise} \end{cases}.$$ 7

13. Find the Fourier sine transform of

$$f(x) = \frac{e^{-ax}}{x}.$$ 7

14. State and prove convolution theorem on Fourier transform, i.e.,

$$F[f(x) * g(x)] = F[f(x)] \cdot F[g(x)].$$ 7

15. Find the function whose sine transform is

$$\frac{e^{-as}}{s}.$$ 7
