BME-015

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

December, 2017

00072

BME-015 : ENGINEERING MATHEMATICS – II

Time : 3 hours *Maximum Marks : 70*

Note : *Answer any **ten** questions. All questions carry equal marks. Use of scientific calculator is permitted.*

---

1. Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < a,\ 0 < y < b,$$

subject to the boundary conditions

$u(0, y) = u(a, y) = 0$, $u(x, b) = 0$, and $u(x, 0) = f(x)$. 7

2. In the vibrating string problem an elastic string of length l is fixed at $x = 0$, and at $x = l$. It is taken to the position $f(x) = A \sin\frac{2\pi x}{l}$ at $t = 0$ and then released. Find the displacement function of the string motion. 7

3. Solve : 7

$$(D^2 + 3DD' + 2D'^2)\, z = x + y$$

4. Solve : 7

$$(D^2 + 2DD' + D'^2)\, z = e^{2x + 3y}$$

5. Solve : 7

$$x\,(y^2 - z^2)\, p + y\,(z^2 - x^2)\, q = z\,(x^2 - y^2)$$

6. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $[-\pi, \pi]$. Also deduce that

$$\frac{1}{1\cdot3} - \frac{1}{3\cdot5} + \frac{1}{5\cdot7} - \frac{1}{7\cdot9} + \ldots = \frac{1}{4}(\pi - 2).$$ 7

7. Find the Fourier series for the function

$$f(x) = \begin{cases} x, & -1 < x \leq 0 \\ x + 2, & 0 < x < 1 \end{cases}$$

where $f(x) = f(x + 2)$. From the series obtained, deduce the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$ 7

8. Find the residue of the following function at each pole : 7

$$f(z) = \frac{z^2 + 1}{z^2 - 2z}$$

9. Expand

$$f(z) = \frac{7z - 2}{z(z+1)(z-2)}$$

as a Laurent series in the region $1 < |z + 1| < 3$. 7

10. Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. 7

11. Test the convergence of the series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots \infty.$$ 7

12. Test the convergence of the series

$$\sum \left[\sqrt[3]{n^3 + 1} - n\right].$$ 7

13. Show that the polar forms of the Cauchy-Reimann equation are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}.$$

Also deduce that

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0.$$ 7

14. Prove that

$$\int_C \frac{dz}{z - a} = 2\pi i$$

where C is the circle $|z - a| = r$. 7

15. If $2 \cos \theta = x + \frac{1}{x}$ and $2 \cos \phi = y + \frac{1}{y}$, show that one of the values of

$$x^m y^n + \frac{1}{x^m y^n} \text{ is } 2 \cos (m\theta + n\phi).$$ 7