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BME-001

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

00202

Term-End Examination

December, 2017

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of calculator is allowed.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Show that the following functions are one-one and onto and find their inverse :

(i) $\frac{x^2}{5} + 2$

(ii) $\frac{x+1}{x-1}$

(b) Which of the following functions are odd, which are even or which are neither odd nor even ?

(i) $f(x) = x^3$

(ii) $f(x) = \begin{cases} 0, \text{ if } x \text{ is rational} \\ 1, \text{ if } x \text{ is irrational} \end{cases}$

(iii) $f(x) = x^2 - 1$

(iv) $f(x) = x^2 + x^3$

(c) Evaluate the limits that exist :

(i) $\underset{x \to 3}{\text{Lt}} \; [(x^2 + x - 12)^2/(x - 3)]$

(ii) $\underset{x \to 0}{\text{Lt}} \; [\sin^2(x)/x(1 - \cos x)]$

(d) Find the value of b for which the function

$$f(x) = \begin{cases} x^2 + 1 & \text{when } x < 2 \\ bx + \dfrac{2}{x} & \text{when } x \geq 2 \end{cases}$$

is continuous at $x = 2$.

(e) Expand the polynomial

$$f(x) = x^5 - 2x^4 + x^3 - x^2 + 2x - 1$$

in the power of $(x - 1)$, using Taylor's formula.

(f) Integrate ***one*** of the following :

(i) $\displaystyle\int \frac{dx}{(e^x + e^{-x})^2}$

(ii) $\displaystyle\int \frac{x^2 + x - 1}{(x - 1)(x^2 - x + 1)} dx$

(g) Solve ***one*** of the following differential equations :

(i) $(2x + y + 1)\, dx + (4x + 2y - 1)\, dy = 0$

(ii) $\dfrac{dy}{dx} + \dfrac{2x}{x^2 - 1} y = e^x$

(h) Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and x – ax is in the first quadrant.

2. Answer any ***four*** of the following : *4×5=20*

(a) Show that the points with the position vectors

$$2\hat{i} + 3\hat{j}, \ \ 3\hat{i} + \frac{9}{4}\hat{j} \text{ and } 5\hat{i} + 0{\cdot}75\hat{j}$$

are collinear.

(b) Prove that :

$$(\vec{a} + \vec{b}) \cdot [(\vec{b} + \vec{c}) \times (\vec{c} \times \vec{a})] = 2[\vec{a}, \vec{b}, \vec{c}]$$

(c) Find the directional derivative of

$$[x^2 + y^2 + 4xyz] \text{ at } (1, -2, 2)$$

in the direction of $(2\hat{i} - 2\hat{j} + \hat{k})$.

(d) Determine the electric field, $\vec{E} = -\nabla\phi$, and charge distribution, $\rho = \varepsilon \nabla \cdot \vec{E}$, corresponding to the potential $\phi = \alpha r^2$.

(e) If $\vec{F} = (3x^2 + 6xy)\,\hat{i} - 14yz\,\hat{j} + 20x^2z\,\hat{k}$, evaluate the line integral $\int \vec{F} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t$, $y = t^2$, $z = t^3$.

(f) Evaluate $\oint_C (x^2 + xy)\,dx + (x^2 + y^2)\,dy$, where C is the boundary of $y = \pm 1$, $x = \pm 1$.

3. Answer any ***five*** of the following : *5×3=15*

(a) (i) Let $T : R^2 \rightarrow R^3$ be a linear transformation such that $T(1, 1) = (0, 1, 1)$ and $T(0, 1) = (1, 0, 1)$. Determine $T(2, 3)$.

(ii) {(1, 0, 0), (1, 1, 0), (1, 1, 1)} is a basis of R^3. Is {(1, 0, 0), (1, 1, 0), (4, 5, 0)} a basis of R^3 ?

(b) Verify that the matrix

$$A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

is orthogonal.

(c) Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} =$$

$$(a-b)(b-c)(c-a)(a+b+c).$$

(d) If $A = \begin{bmatrix} 1 & -\tan x & 0 \\ \tan x & 1 & 0 \\ 0 & 0 & \sec x \end{bmatrix}$,

show that

$$A^{-1} = \cos x \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(e) Find the rank of the matrix

$$\begin{bmatrix} 2 & -3 & 6 & -5 \\ 0 & 1 & -4 & 1 \\ 4 & -5 & 8 & -9 \end{bmatrix}.$$

(f) Find the eigenvectors for the matrix

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}.$$

(g) Find k, l, m to make A as a Hermitian matrix.

$$A = \begin{bmatrix} -1 & k & -i \\ 3-5i & 0 & m \\ l & 2+4i & 2 \end{bmatrix}$$

(h) Find the characteristic roots of the matrix

$$A = \begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}.$$

4. Answer any ***three*** of the following : *3×5=15*

(a) The probability that a man aged 60 will live up to 70 is 0·65. What is the probability that out of 10 men, aged 60, at least 7 will live to be 70 ?

(b) If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random,

(i) 1

(ii) 0

(iii) at least 2

will be defective.

(c) Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percentage of students scored more than 60 marks ?

(d) Customers arrive in a bank at an average rate of two, every 10 minutes. The number of arrivals is distributed according to a Poisson distribution. What is the probability that there will be

(i) no arrival during any period of ten minutes ?

(ii) exactly one arrival during this time period ?

(iii) more than two arrivals during this time period ?

(Use $e^{-2} = 0{\cdot}1353$)

(e) Nine items of a sample have the following values :

45, 47, 52, 48, 47, 49, 53, 51, 50

Does the mean of nine items differ significantly from the assumed population mean of 47·5 ?

Given that for degree of freedom = 8, P = 0·945 for t = 1·8 and P = 0·953 for t = 1·9.

(f) A machine produced 16 defective articles in a batch of 500. After overhauling, it produced 3 defective articles in a batch of 100. Has the machine improved significantly ?