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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

00197 **December, 2017**

ET-102 : MATHEMATICS – III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any* ***ten*** *questions. All questions carry equal marks. Use of calculator is allowed.*

1. Test the convergence of the series

$$\frac{x}{2}+\frac{x^2}{5}+\ldots+\frac{x^n}{n^2+1}+\ldots$$

for all values of x. 7

2. Show that the series

$$\sum_n \frac{(-1)^{n-1}}{n^p}$$

converges conditionally for $0 < p \leq 1$. 7

3. Find half-range sine series for the function

$f(x) = x(\pi - x)$ for $0 \leq x \leq \pi$. 7

4. If $w = \phi + i\,\psi$ represents the complex potential for an electric field and

$$\psi = x^2 - y^2 + \frac{1}{x^2 + y^2},$$

determine the function ϕ. 7

5. Find the Laurent expansion of the function

$$f(z) = \frac{7z - 2}{(z + 1)\, z\, (z + 2)}$$

in the annulus $|z + 1| > 3$. 7

6. Determine all the singularities and residues thereat of the function

$f(z) = (z^n\, e^{1/z})/(1 + z)$, $n \in N$. 7

7. By integrating $[(e^{iz})/z]$ around a suitable contour, prove that

$$\int_0^{\infty} \frac{\sin x}{x}\, dx = \pi/2.$$ 7

8. For the conformal transformation $w = z^2$, show that

(i) Coefficient of magnification at $z = 2 + i$ is $2\sqrt{5}$.

(ii) The angle of relation at $z = 2 + i$ is $\tan^{-1}(0{\cdot}5)$.

(iii) The circle $|z - 1| = 1$ transforms into the cardioid $\rho = z(1 + \cos\theta)$, where $w = \rho e^{i\theta}$ in the w-plane. 7

9. Evaluate : 7

$$\mathcal{L}^{-1}\left[\frac{3s - 2}{s^3(s^2 + 4)}\right]$$

10. Using Laplace transform, solve the differential equation

$$y'' - 6y' + 9y = t^2 e^{3t}, \text{ with } y(0) = 2, y'(0) = 6. \quad 7$$

11. Use the tabular form of Hurwitz-Routh criterion for stability of the differential equation whose characteristic equation is

$$s^4 + 6s^3 + 16s^2 + 16s + 5 = 0. \quad 7$$

12. Solve : 7

$$\left(x \sin\frac{y}{x}\right) dy = \left(y \sin\frac{y}{x} - x\right) dx$$

13. Reduce the equation

$$(2x - 10y^2)\, dy + y\, dx = 0$$

to the linear form and find its solution. 7

14. Solve

$(D^2 + 2)\, y = x^2\, e^{3x}$, where $D \equiv \frac{d}{dx}$. 7

15. Solve

$$(z - y)\, p + (x - z)\, q = y - x,$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 7