ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

00547

December, 2017

ET-101(A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of scientific calculator is allowed.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate :

$$\lim_{x \to 0} \frac{1 - \cos x}{x \sin x}$$

(b) If $x = \cos (ln\, y)$,

show that $(1 - x^2)\, y_2 - xy_1 = y$.

(c) If $y = a \sin (m \sin^{-1} x)$,

prove that

$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} + (m^2 - n^2)\, y_n = 0.$

(d) If f(x) be a function of real variable x, and f(x) defined by

$$f(x) = -x, \quad \text{when } x \le 0$$
$$= x, \quad \text{when } 0 < x < 1$$
$$= 2 - x, \quad \text{when } x \ge 1,$$

show that f(x) is continuous at x = 0 and also at x = 1.

(e) If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$,

find

$$\frac{\partial(x, y, z)}{\partial(r, \theta, z)}.$$

(f) If $\sin y = x \sin(a + y)$,

prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}.$$

(g) The horsepower (H) developed by an aircraft travelling horizontally with velocity V m/s is

$$H = \frac{AW^2}{V} + BV^2,$$

where A, B and W are constants, W representing the weight of the aircraft.

Find for what value of V the horsepower is minimum.

(h) Show that the semi-vertical angle of the cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$.

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate (any ***one***) :

(i) $$\int \frac{1}{(3+x)\sqrt{1+x}}\,dx$$

(ii) $$\int \frac{x^3}{1+x^8}\,dx$$

(b) Evaluate (any ***one***) :

(i) $$\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x}\,dx$$

(ii) $$\int_1^2 x^2 \log x\,dx$$

(c) Find the area included between the parabola $y^2 = 4ax$ and its latus rectum.

(d) Find the surface area of a cone generated by the revolution of a line segment $y = 2x$ from $x = 0$ to $x = 2$ about the x-axis.

(e) Find the volume generated by revolving the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, about the x-axis.

(f) Solve (any ***one***) :

(i) $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

(ii) $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$

3. Answer any ***four*** of the following : *4×4=16*

(a) A particle moves along the curve $x = 4 \cos t$, $y = 4 \sin t$ and $z = 6t$. Find its velocity and acceleration at $t = 0$ and $t = \pi/2$.

(b) Find the directional derivative of the function $\phi = xyz^2$ in the direction of the vector $2i + j - k$ at the point (2, 3, 1).

(c) If $f(x, y, z) = 3x^2y - y^3z^2$, then find ∇f at the point (1, – 2, – 1).

(d) If $\vec{\mathbf{r}} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that $\text{div}\,\vec{\mathbf{r}} = 3$.

(e) If $\vec{\mathbf{F}} = (x^2 - y^2)\,\hat{i} + 2xy\,\hat{j} + (y^2 - xy)\,\hat{k}$, then find $\nabla \cdot \vec{\mathbf{F}}$.

(f) A fluid motion is given by $\vec{\mathbf{q}} = (y + z)\,\hat{i} + (z + x)\,\hat{j} + (x + y)\,\hat{k}$. Is this motion irrotational ? If so, find the velocity potential. Is the motion possible for an incompressible fluid ?

4. Answer any ***six*** of the following : *6×3=18*

(a) Find x, y, z and t, so that

$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}.$$

(b) If

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix},$$

compute the product AB. Can the product be computed ? If possible, find BA.

(c) Find the inverse of

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 3 & -1 & -4 \end{bmatrix}$$

and hence solve the equations :

$$2x + y + z = 11$$

$$x - 2y - z = -8$$

$$3x - y - 4z = -13$$

(d) Find the rank of the matrix

$$A = \begin{bmatrix} 6 & 1 & 3 \\ 4 & 2 & 6 \\ 10 & 3 & 9 \end{bmatrix}.$$

(e) Show that

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} =$$

$$\begin{bmatrix} 1 & -\tan\dfrac{\theta}{2} \\ \tan\dfrac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\dfrac{\theta}{2} \\ -\tan\dfrac{\theta}{2} & 1 \end{bmatrix}^{-1}$$

(f) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{bmatrix}.$$

(g) Verify that

$$\frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$$

is an orthogonal matrix.

(h) If

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

choose α and β so that $[\alpha I + \beta A]^2 = A$.

Find α and β, where I is the unit matrix.