No. of Printed Pages : 5

MST-003

POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

01074 **December, 2016**

MST-003 : PROBABILITY THEORY

Time : 3 hours *Maximum Marks : 50*

Note :

*(i) Attempt **all** questions. Questions no. 2 to 5 have internal choices.*

(ii) Use of scientific calculator is allowed.

(iii) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.

(iv) Symbols have their usual meaning.

---

1. State whether the following statements are *True* or *False* ? Give reasons in support of your answers.

5×2=10

(a) If $P(A) = \frac{2}{7}$, then the odds in favour of A are 2 : 7.

(b) Probability of an impossible event is always greater than zero.

(c) If $X \sim N(\theta, \sigma^2)$ and $Z = \frac{X - \theta}{\sigma}$, then the variance of Z is 0.

(d) There exists a geometric distribution having mean 4 and variance 3.

(e) If $E(X) = 5$, then $E(Y) = 29$, where $Y = 2X + 3$.

2. (a) Out of 52 well-shuffled playing cards, two cards are drawn at random. Find the probability of getting : 5

(i) One red and one black

(ii) Both cards of the same suit

(iii) One jack and one king

(iv) One red and one club

(b) A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A), given that $P(B) = \frac{3}{4} P(A)$ and $P(C) = \frac{1}{3} P(B)$. 5

OR

(a) A person X speaks the truth in 80% cases and another person Y speaks the truth in 90% cases. Find the probability that they contradict each other in stating the same fact. 5

(b) An insurance company insured 1000 scooter drivers, 3000 car drivers and 6000 truck drivers. The probabilities that the scooter, car and truck drivers meet an accident are 0·02, 0·04 and 0·25, respectively. One of the insured persons meets with an accident. What is the probability that he is a car driver ? 5

3. (a) The p.d.f. of the different weights of "one litre pure ghee pack" of a company is given by

$$f(x) = \begin{cases} 200(x-1) & \text{for } 1 \le x \le 1{\cdot}1 \\ 0, & \text{otherwise} \end{cases}$$

Examine whether the given p.d.f. is a valid one. If yes, find the probability that the weight of any pack will lie between 1·01 and 1·02. 5

(b) The point probability distribution of a pair of random variables is given by the following table :

X \ Y	1	2	3
1	1/12	0	1/18
2	1/6	1/9	1/4
3	0	1/5	2/15

Obtain the joint and marginal distribution functions. 5

OR

(a) Let X and Y be two random variables. Then for

$$f(x, y) = \begin{cases} k(2x + y), & 0 < x < 1, \quad 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

to be a joint density function, what must the value of k be ? 5

(b) Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the expected value for the number of aces. 5

4. (a) The probability of a man hitting a target is $\frac{1}{4}$. He tries 5 times. What is the probability of his hitting the target at least twice ? 5

(b) If the probability that an individual suffers a bad reaction from an injection of a given serum is 0·001, determine the probability that out of 1500 individuals (i) exactly 3, and (ii) more than 2 individuals suffer from a bad reaction. 5

OR

(a) Let us suppose that in a lake there are N fish. A catch of 500 fish (all at the same time) is made and these fish are returned alive into the lake after marking each with a red spot. After two days, assume that during this time these 'marked' fish have distributed themselves 'at random' in the lake and there is no change in the total number of fish, a fresh catch of 400 fish (again, all at once) is made. What is the probability that of these 400 fish, 100 will be having red spots ? 5

(b) An unbiased die is cast until 6 appears. What is the probability that it must be cast more than 5 times ? 5

5. (a) If X_1 and X_2 are independent variables such that $X_1 \sim N(40, 25)$, $X_2 \sim N(60, 36)$, then find the mean and variance of

(i) $X = 2X_1 + 3X_2$ (ii) $Y = 3X_1 - 2X_2$. *5*

(b) If the random variable X is normally distributed with mean 80 and standard deviation 5, then find $P[64 < X < 76]$. *5*

OR

(a) Metro trains are scheduled every 5 minutes at a certain station. A person comes to the station at a random time. Let the random variable X count the number of minutes he has to wait for the next train. Assume X has a uniform distribution over the interval (0, 5). Find the probability that he has to wait at least 3 minutes for the train. *5*

(b) Determine the constant k such that the function

$$f(x) = \frac{kx^3}{(1+x)^7}, \; 0 < x < \infty,$$

is the p.d.f. of beta distribution of the second kind. Also find its mean and variance. *5*
