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MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00294

Term-End Examination

December, 2016

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Answer any **four** questions from questions no. 1 to 5. Question no. **6** is **compulsory**. Use of calculator is **not** allowed.*

1. (a) Define 'linear code', and give an example with justification. 3

   (b) Prove that a binary repetition code of odd length is a perfect code. 3

   (c) Let $\alpha = x + \mathbf{F}_2[x] / < x^3 + x + 1 >$. Make a table that gives the powers of α as a linear combination of 1, α and α^2. Use it to write $(\alpha^5 - \alpha^2 + 1)(\alpha + 1) / (\alpha^2 + 1)$ as a linear combination of 1, α and α^2. 4

2. (a) Compute the 3-cyclotomic cosets modulo 7. 2

(b) Find the generator matrix and the parity check matrix for the binary cyclic code of length 7 with generator polynomial $(x^3 + x + 1)$. 4

(c) Let C be a cyclic code over $\mathbf{F}_q$ with generating idempotent e(x). Prove that the generator polynomial of C is

$g(x) = \gcd(e(x), x^n - 1)$ computed in $\mathbf{F}_q[x]$. 4

3. (a) Let C be a [15, 7] narrow-sense binary BCH code of designed distance $\delta = 5$, which has the defining set T = {1, 2, 3, 4, 6, 8, 9, 12}. Using the primitive 15th root of unity α, where $\alpha^4 = 1 + \alpha$, the generator polynomial of C is $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. Suppose C is used to transmit a code word and $y(x) = x^6 + x^{10} + x^{12}$ is received. Find the transmitted code word.

You may use the following table : 6

0000	0	1000	α^3	1011	α^7	1110	α^{11}
0001	1	0011	α^4	0101	α^8	1111	α^{12}
0010	α	0110	α^5	1010	α^9	1101	α^{13}
0100	α^2	1100	α^6	0111	α^{10}	1001	α^{14}

(b) Construct the generating idempotents of the duadic codes of length 7 over $\mathbf{F}_4$. 4

4. (a) Let C be the [5, 2] binary code generated by

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

(i) Find the weight distribution of C.

(ii) Find the weight distribution of $C^{\perp}$ by using the Mac-Williams identity. 6

(b) Let C be the $\mathbf{Z}_4$-linear code of length 3 with generator matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}.$$

(i) List the 16 code words in C.

(ii) List the 16 code words in the Gray image of C. 4

5. (a) Let C be the (2, 1) convolutional code with generator matrix $[1 \quad 1 + D]$. Prove that the code C has free distance 3. 6

(b) (i) Explain the Tanner graph of a code. 2

(ii) Let C be the [7, 4, 2] binary code with parity-check matrix.

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Give the Tanner graph of this code. 2

6. Which of the following statements are *True*, and which are *False* ? Justify your answers. *10*

(a) $5^{10} \equiv 1 (\text{mod } 10)$.

(b) If C is an [n, k]-code with parity-check matrix P, then any two words x, y in C have the same syndrome only if x = y.

(c) If x and y are two code words in an LDPC code, with the minimum distance between them being less than 1, then x and y will differ in only one component.

(d) The dimension of a code C is the same as the dimension of the dual of C.

(e) The minimum distance of any code is its error-coding capability.