No. of Printed Pages : 4

MMTE-001

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00534

December, 2016

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is **compulsory**. Answer any **four** out of the remaining five (questions no. 2 to 6) questions. Calculators are **not** allowed.*

1. State, with justification or illustration whether each of the following statements [(a) to (e)] is *True* or *False*. *5×2=10*

   (a) Every graph with n vertices and k edges has at least n – k components.

   (b) There are graphs G with diam G = rad G.

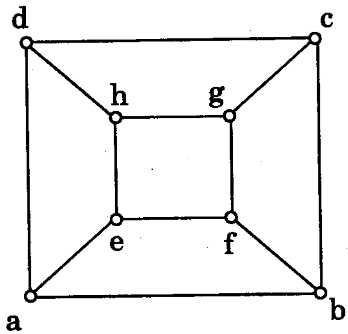
   (c) Every complete graph has a perfect matching.

   (d) If G is a graph with two non-adjacent vertices u and v and G + uv is Hamiltonian, then G is Hamiltonian.

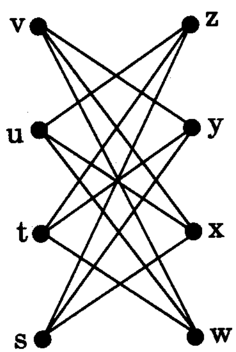
   (e) If G is a simple graph with $\zeta(G) \leq 3n(G) - 6$, then G is planar.

2. (a) Determine which pairs of graphs given below are isomorphic. Justify your answer. *4*

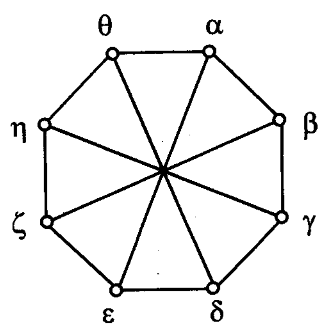
(i)



(ii)



(iii)



(b) Prove that an edge e in a graph G is a cut-edge if and only if e belongs to no cycle in G. *3*

(c) Prove that every even graph decomposes into cycles. *3*

3. (a) Prove that if a set $\{d_1, d_2, d_3, \ldots d_n\}$ of non-negative integers represents the degrees of vertices in a graph then, $\sum_{i=1}^{n} d_i$ is even and $d_i \leq n - 1$, for each $i = 1, \ldots, n$. Is the converse true ? Justify your answer. *4*

(b) Find the values of n for which $K_{n, n}$ is Hamiltonian. Justify your answer. *3*

(c) If G is a self-complementary graph of order $n > 1$, then prove that *diam* $(G) = 3$. *3*

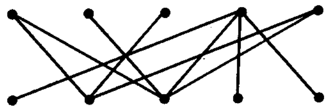
4. (a) The number of leaves in any tree of order n,

$$n \geq 1, \text{ is } 1 + \frac{1}{2} \sum_{v \in V(G)} |d(v) - 2|.$$ *4*

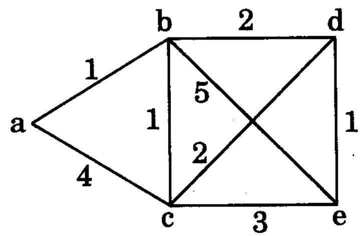
(b) For every graph G, prove that

$$\chi(G) \cdot \alpha(G) \geq n(G).$$ *3*

(c) Find a maximum matching in the following graph. Justify your answer. *3*



5. (a) Use Dijkstra's algorithm to find the shortest paths from vertex a to all other vertices, sharing all the necessary steps. *4*



(b) For any graph G, prove that

$$\alpha(G) + \beta(G) = n(G).$$ *3*

(c) Construct a simple cubic graph having no 1-factor. *3*

6. (a) Prove that $K(G) \leq K'(G) \leq \delta(G)$ for any simple graph G and construct a graph for which $K(G) = 1$, $K'(G) = 2$ and $\delta(G) = 3$. *6*

(b) Using Mycielski's construction, produce a 4-chromatic graph from C_5. *4*
