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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00434

December, 2016

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any **five** questions. Use of calculator is **not** allowed.*

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1. (a) Let P(t), measured in kg, be the total mass or biomass of the fish population in a pond at time t. Write the continuous model for the population growth using logistic equation. The intrinsic growth rate r and the carrying capacity k are given the values 0·71 per year and $80{\cdot}5 \times 10^6$ kg, respectively. If the initial biomass is $P_0 = 0{\cdot}25$ k, find the biomass after 2 years. Also find the time t_1 for which $P(t_1) = 0{\cdot}75$ k. *3*

   (b) Indifference curves of an investor cannot intersect. Is this statement true ? Give reasons for your answer. *2*

2. Consider the delay model of a population growth given by the difference equation

$$u_{n+1} = u_n \exp\left[r\left(2 - \frac{u_{n-1}}{2}\right)\right], \; r > 0.$$

Find the linear steady-states of the model and do the stability analysis when $0 < r < \frac{1}{8}$. 5

3. Consider the following prey-predator model under toxicant stress :

$$\frac{dN_1}{dt} = r_0 N_1 - r_1 C_0 N_1 - bN_1 N_2$$

$$\frac{dN_2}{dt} = -d_0 N_2 + \beta_0 \, bN_1 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - m_1 C_0$$

$$\frac{dP}{dt} = Q - hP - kPN_1 + g_1 C_0 N_1$$

under the conditions

$N_1(0) = N_{10}, N_2(0) = N_{20}, C_0(0) = 0, P(0) = P_0 > 0,$

where $N_1(t)$ = Density of prey population,

$N_2(t)$ = Density of predator population,

$C_0(t)$ = Concentration of toxicant in the individuals of the prey population,

r_0 = growth rate, d_0 = death rate,

b = predation rate, β_0 = conversion coefficient,

m_1 = depuration rate, k, k_1 = uptake rates,

g_1 = loss rate, r_1 = death rate due to C_0,

Q = rate of toxicant entering into the environment,

P = environmental toxicant concentration.

Reformulate the above model if the environmental toxicant concentration is assumed to be a constant. Do the stability analysis of the reformulated model. 5

4. The yearly fluctuations in the ground water table is believed to be dependent on the annual rainfall and the volume of water pumped out from the basin. The data collected on these variables for four consecutive years is given below :

Water table (in cm)	Annual rainfall (in cm)	Ground water volume pumped out (in cm^3)
10	3	7
9	4	8
7	5	9
4	7	7

Use the method of least squares to find a linear regression equation that best fits the data. 5

5. Assume that the return distribution of two securities, X and Y, of portfolio P be as given below :

Possible rates of returns of security		Associated Probabilities
X	Y	$p_{xj} = p_{yj}$
0·11	0·18	0·42
0·17	0·16	0·15
0·10	0·11	0·30
0·19	0·09	0·13

Find the correlation coefficient between the securities X and Y. If the portfolio weights are 50% each in X and Y, then find the risk of the portfolio P. *5*

6. Solve an optimal transportation problem so that the cost of transporting 340 tons of coal from three mines A, B and C to five power plants I, II, III, IV and V is minimized. The five power plants must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons of coal, respectively. The availability of coal at mines A, B and C is 100 tons, 120 tons and 120 tons, respectively. The transportation cost per ton from mines to power plants is given below : *5*

	I	II	III	IV	V
A	4	1	2	6	9
B	6	4	3	5	7
C	5	2	6	4	8