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MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00434

December, 2016

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***four*** *questions out of the remaining questions no. 2 to 7. All computations may be kept to three decimal places. Use of calculators is* ***not*** *allowed.*

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1. State whether the following statements are *True* or *False*. Justify your answers with the help of a short proof or a counter-example. *5×2=10*

   (a) For the differential equation

   $x^2(x-4)^2\, y''(x) + 3xy'(x) - (x-4)\, y = 0,$

   $x = 0$, is a regular singular point and

   $x = 4$, is an irregular singular point.

   (b) x^2 when expanded into a series of Legendre's polynomial, yields

   $$\frac{2}{3} P_2(x) + \frac{1}{2} P_0(x).$$

   (c) The Runge-Kutta method of second order is nothing but the modified Euler's method.

(d) The Laplace transform of the following convolution integral $\int_0^t (t-\beta)^3 e^{\beta} d\beta$ equals $[6/s^4(s+1)]$.

(e) The explicit scheme;

$$u_i^{n+1} = u_i^n + \lambda[u_{i+1}^n - 2u_i^n + u_{i-1}^n]$$

where $\lambda = k/h^2$, for solving the parabolic equation $u_t = u_{xx}$ is stable for $\lambda < 1$.

2. (a) Using Laplace transform technique, solve the following initial value problem : 6

$$\frac{dx}{dt} + \frac{dy}{dt} = t, \quad \frac{d^2x}{dt^2} - y = e^{-t},$$

$$x(0) = 0,\ y(0) = 0,\ \frac{dx}{dt} = 0, \text{ for } t = 0.$$

(b) Using the recurrence relation

$$(n+1)P_{n+1}(x) = (2n+1)\, xP_n(x) - nP_{n-1}(x),$$

prove that

$$(2n+1)^2 \int_{-1}^{1} x^2 P_n^2(x)\, dx = \frac{2(n+1)^2}{(2n+3)} + \frac{2n^2}{(2n-1)}.$$ 4

3. (a) Solve the following differential equation by power series method about $x = 0$: 5

$$(1 - x^2)\, y''(x) - 2xy'(x) + 2y(x) = 0.$$

(b) Find the Fourier sine transform of 5

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2. \\ 0, & x > 2 \end{cases}$$

4. (a) Given $\frac{dy}{dx} = x - y^2, y(0{\cdot}2) = (0{\cdot}02)$. Find $y(0{\cdot}4)$ by using modified Euler's method, correct to two decimal places, taking $h = 0{\cdot}2$. 5

(b) Determine an appropriate Green's function either by using the method of variation of parameters or otherwise, for the following boundary value problem : 5

$$-(y''(x) - y(x)) = \frac{2}{1 + e^{-x}}, \; y(0) = y(1) = 0.$$

5. (a) Find the solution of the initial boundary value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 \le x \le 1$

$u(x, 0) = \sin \pi x, 0 \le x \le 1$

$\frac{\partial u}{\partial t}(x, 0) = 0,$

$u(0, t) = u(1, t) = 0, t > 0$ by using second order explicit method with $h = \frac{1}{4}, r = \frac{1}{3}$.

Integrate for one time step. 4

(b) Prove that

$$\frac{d}{dx}(J_n^2(x)) = \frac{x}{2n}[J_{n-1}^2(x) - J_{n+1}^2(x)].$$ 4

(c) Given

$$\frac{dy}{dx} = -300y, \; y(0) = 1.$$

Determine the value of h so that the second order Runge-Kutta method applied to the IVP produces stable results. 2

6. (a) Find the Laplace inverse transform of

$$F(s) = ln\left(1 + \frac{1}{s^2}\right).$$ 4

(b) Using Milne's fourth order predictor-corrector method find y(2), given

$$\frac{dy}{dx} = \frac{1}{2}(x + y), \; y(0) = 2,$$

where y(0·5) = 2·636, y(1) = 3·595,

y(1·5) = 4·968. Perform two corrector iterations. 6

7. (a) Solve the Poisson's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2x^2 y^2$$

over the square mesh domain $0 \le x \le 3$ and $0 \le y \le 3$ with $u = 0$, on the boundary and $h = k = 1$. Use five-point formula. *6*

(b) For the boundary value problem

$$\frac{d^2 y}{dx^2} = e^{x^2}; \; y(0) = 0, \; y(1) = 0,$$

estimate, using second order finite difference method the values of $y(x)$ at $x = 0{\cdot}25, 0{\cdot}5$ and $0{\cdot}75$

(given $e^{\left(\frac{1}{2}\right)^2} = 1{\cdot}2840$, $e^{\left(\frac{1}{4}\right)^2} = 1{\cdot}0645$,
$e^{\left(\frac{3}{4}\right)^2} = 1{\cdot}7551$). *4*