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MMT-003

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)
Term-End Examination

00834

December, 2016

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

(Weightage : 70%)

Note : *Question no. 1 is* **compulsory**. *Answer any* **four** *questions from questions no. 2 to 6.*

1. State, with reasons, which of the following statements are *True* and which are *False* : 10

(a) If G is a group containing normal subgroups of order 3 and 5, then G contains an element of order 15.

(b) There is a finite field of order 12.

(c) Every free group is abelian.

(d) $\begin{bmatrix} -1 & 1 \\ 3 & 3 \end{bmatrix}$ is a symplectic matrix.

(e) $\rho : S_3 \to S_3 : \rho(x) = x$ is a representation of S_3.

2. (a) Find the degree of $\mathbf{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbf{Q}$. *4*

(b) Define a regular language. Give an example of the same, with justification. *3*

(c) Show that $\rho(e^{i\theta}) = \begin{bmatrix} e^{i\theta} & e^{2i\theta} - e^{i\theta} \\ 0 & e^{2i\theta} \end{bmatrix}$ is a representation of $\{e^{i\theta} \mid \theta \in \mathbf{R}\}$. Is the representation unitary ? Give reasons for your answer. *3*

3. (a) Prove that if $p > 2$ is a prime, then $(1\ \ 2)$ and $(1\ \ 2 \ldots p-1\ \ p)$ generate S_p. *5*

(b) Prove that if $K \subseteq E \subseteq L$ are fields such that L/K is a normal extension, then L/E is also normal. *3*

(c) If $G = \{i, (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 8), (1\ 3\ 2)\ (4\ 6\ 5), (1\ 2\ 3)\ (4\ 5\ 6), (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8), (7\ 8)\}$, find the stabiliser of 7 in G, where G acts on $\{1, 2, \ldots 6, 7, 8\}$ as permutations. *2*

4. (a) Find an integer x such that

$2x \equiv 1 \pmod 3$,

$3x \equiv 1 \pmod 5$,

$5x \equiv 1 \pmod 7$ simultaneously. *5*

(b) Consider the incomplete character table, for the tetrahedral group given below, in which all the conjugacy classes are given :

	(1) x_1	(3) x_2	(4) x_3	(4) x_4
χ_1	1	1	1	1
χ_2	1	1	ω	ω^2
χ_3	1	1	ω^2	ω

where ω is a primitive cube root of unity.

(i) What is the order of the group ?

(ii) How many characters are missing ?

(iii) Find the missing characters and complete the table.

(iv) Find the order of the kernel of the representation(s) corresponding to the missing character(s). *5*

5. (a) Check whether the ISBN number 0-387-97329-X is a valid ISBN number. *3*

(b) Let P be a matrix in $SO_3(\mathbf{C})$. Then prove that 1 is an eigenvalue of P. *4*

(c) Consider the binary linear code C = {0000, 0001, 0011, 0111, 0010, 0110, 0100, 0101}. Find a generator matrix for C. Is this generator matrix in systematic form ? Justify your answer. *3*

6\. (a) State the structure theorem for finitely generated abelian groups. Further, find the invariant factors of the group $\mathbf{Z}_{49} \times \mathbf{Z}_{28}$. 4

(b) Let $\mathbf{F}$ be a finite field with q elements and $A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \in GL_2(\mathbf{F})$. Find N(A). 3

(c) Give an example, with justification, of a group action on a non-empty set S. 3