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MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

01115 **December, 2016**

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no. 5 is **compulsory**. Answer any **three** questions from questions no. 1 to 4. Calculators are **not** allowed.*

1. (a) Show that the matrix $A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{pmatrix}$ is nilpotent. *1*

   (b) Let A be the matrix in question 1(a). Compute the solution of the system of differential equations

   $$\frac{dy(t)}{dt} = A\,y(t) \text{ with } y(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$ *4*

2. (a) Use the least squares method to find the line that best fits the data

   (–1, 0), (0, 1), (1, 2), (2, 4). *3*

(b) Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be defined by

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y + 3z \\ x + 2y - z \end{bmatrix}.$$

Find the matrix of T with respect to the bases $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ of $\mathbf{R}^3$ and $\mathbf{R}^2$, respectively. *2*

3\. (a) Show that $\begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix}$ is positive definite and find its square root. *3*

(b) Check whether the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ is diagonalisable. *2*

4\. (a) Find the singular value decomposition of

$$A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}.$$ *3*

(b) Give an example, with justification, of two matrices whose characteristic and minimal polynomials are the same, but their Jordan forms are different. *2*

5. State whether the following statements are *True* or *False*. Give reasons for your answer. *5×2=10*

(a) If x is an $n \times 1$ vector of norm 1, then $x^+ = x^*$.

(b) Every orthogonal matrix is also a unitary matrix.

(c) Any 3×3 matrix with minimal polynomial x^3 is similar to the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

(d) Suppose a matrix A satisfies the equation $(A - I)(A - 2I)(A - 3I) = 0$. Then it is diagonalisable.

(e) If A and B are similar to the identity matrix I_n, then $A = B$.