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B.Tech. – VIEP – MECHANICAL ENGINEERING / B.Tech. CIVIL ENGINEERING (BTMEVI / BTCLEVI)

00493

Term-End Examination

December, 2016

BICE-027 : MATHEMATICS-III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any* ***ten*** *questions. All questions carry equal marks. Use of scientific calculator is permitted.*

1. Expand

$$f(x) = x \sin x,\ 0 < x < 2\pi$$

as a Fourier series. 7

2. Find the Fourier series for the function

$$f(x) = x + x^2 \text{ in the interval } -\pi < x < \pi.$$

Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}.$$ 7

3. Find the Fourier expansion for the function

$$f(x) = x - x^2, \; -1 < x < 1.$$ 7

4. If $f(x) = x, \quad 0 < x < \frac{\pi}{2}$

$= \pi - x, \quad \frac{\pi}{2} < x < \pi,$

show that

$$f(x) = \frac{4}{\pi}\left(\sin x - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots\right).$$ 7

5. Analyse harmonically the data given below and express y in Fourier series up to the third harmonic : 7

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1·0	1·4	1·9	1·7	1·5	1·2	1·0

6. Solve : 7

$$\cos(x + y)\, dy = dx$$

7. Solve : 7

$$\frac{dy}{dx} + y \cot x = \cos x$$

8. Solve : 7

$$\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$$

9. Solve : 7

$$(x^2 - yz)\, p + (y^2 - zx)\, q = z^2 - xy$$

10. Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,$$

where $u(x, 0) = 6e^{-3x}$. 7

11. Find the solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

such that

$$y = P_0 \cos pt,$$

P_0 is a constant when $x = 1$, and $y = 0$ when $x = 0$. 7

12. A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form

$$y = k(lx - x^2)$$

from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t. 7

13. Find the solution of

$$\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$$

for which $u(0, t) = u(l, t) = 0$,

$$u(x, 0) = \sin \frac{\pi x}{l}$$

by the method of variables separable. 7

14. Solve

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \text{ given that}$$

(a) $u = 0$, when $x = 0$ and $x = l$ for all l

(b) $u = 3 \sin \frac{\pi x}{l}$, when $t = 0$ for all x, $0 < x < l$. 7

15. Find by the method of separation of variables the solution of $U(x, t)$ of the boundary value problem

$$\frac{\partial U}{\partial t} = 3 \frac{\partial^2 U}{\partial x^2}, \qquad t > 0, 0 < x < 2$$

$$U(0, t) = 0, \qquad U(2, t) = 0$$

$$U(x, 0) = x, \qquad 0 < x < 2.$$ 7