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BME-001

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

00553

December, 2016

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of scientific calculator is allowed. Statistical tables are allowed.*

1. Attempt any ***five*** of the following : *5×4=20*

(a) Evaluate the limit $\lim_{x \to -3} \dfrac{x^3 - 27}{x + 3}$.

(b) Find $\dfrac{dy}{dx}$, if $y = (\sin x)^x$.

(c) Evaluate $\displaystyle\int_0^1 \frac{\tan^{-1} x}{1 + x^2} dx$.

(d) Use Cauchy's theorem to show that $1 + x < e^x$.

(e) If $u = x + y + z$, $y + z = uv$, $z = uvw$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

(f) Solve the differential equation

$(1 - \sin x \tan y)\, dx + (\cos x \sec x^2)\, dy = 0.$

2. Attempt any ***four*** of the following : *4×5=20*

(a) The position vector of a moving particle is given by $\vec{r}(t) = t^3 \hat{i} + t \hat{j} + t^2 \hat{k}$. Determine the velocity and acceleration of the particle in the direction of the motion.

(b) Find the directional derivative of $f(x, y, z) = xy^2 + 4xyz + z^2$ at the point (1, 2, 3) in the direction of $3\hat{i} + 4\hat{j} - 5\hat{k}$.

(c) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, then prove that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$.

(d) Show that the vector field $\vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$ is irrotational as well as solenoidal.

(e) Evaluate the surface integral

$$\iint_S \vec{F} \cdot \hat{n}\, dS, \text{ where } \vec{F} = z^2\hat{i} + xy\hat{j} - y^2\hat{k}$$

and S is the surface of the cylinder $x^2 + y^2 = 36$, $0 \le z \le 4$ included in the first octant.

(f) Verify Stokes' theorem for $\vec{A} = xz\hat{i} - y\hat{j} + x^2y\hat{k}$, where S is the surface of the region bounded by x = 0, y = 0, z = 0, 2x + y + 2z = 8 which is not included in the xy-plane.

3. Attempt any ***five*** of the following : *5×3=15*

(a) Find the adjoint of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}.$$

(b) Find the inverse of the matrix by elementary transformations.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$

(c) Find the rank of the matrix

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}.$$

(d) Solve the following equations by matrix method :

$x + y - z = 0;\ 2x - y + z = 3;\ 4x + 2y - 2z = 2.$

(e) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

(f) Using the properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha).$$

4. Answer any ***three*** of the following : *3×5=15*

(a) If the events A and B are independent and $P(A) = 0{\cdot}15$, $P(A \cup B) = 0{\cdot}45$, then find $P(B)$.

(b) If a machine is set up correctly, it produces 90% good items; if it is incorrectly set up, then it produces 10% good items. Chances for a setting to be correct and incorrect are in the ratio of 7 : 3. After a setting is made, the first two items produced are found to be good items. What is the chance that the setting was correct ?

(c) The probability that a man aged 60 will live to be 70 is 0·65. What is the probability that out of 10 men, now 60, at least 7 will live to be 70 ?

(d) Ten individuals are chosen at random from the population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches respectively. Discuss the suggestion that the mean height in the universe is 65 inches given that for 9 degrees of freedom the value of student's 't' at 0·5 level of significance is 2·262.

(e) There are 1000 students in a college out of 20000 students in the whole university. In a study, 200 were found smokers in the college and 1000 in the university. Is there a significant difference between the proportion of smokers in the college and in the university ?