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BIEL-023

B.Tech. – VIEP – ELECTRONICS AND COMMUNICATION ENGINEERING (BTECVI)

00923

Term-End Examination

December, 2016

BIEL-023 : INFORMATION THEORY AND CODING

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any* ***seven*** *questions. Assume missing data, if any, suitably. Use of scientific calculator is permitted.*

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1. An information source produces 8 different symbols with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}$ and $\frac{1}{256}$ respectively. These symbols are encoded as 000, 001, 010, 011, 100, 101, 110 and 111 respectively.

   (a) What is the amount of information per symbol ?

   (b) What are the probabilities of a '0' & a '1' occurring ?

   (c) What is the efficiency of the code so obtained ?

   (d) Give an efficient code with the help of the method of Shannon. *10*

2. Let $A = \sum_{n=2}^{\infty} (n \log_n 2)^{-1}$. Show that the integer random variable X defined by

$\Pr(X = n) = (A \cdot n \cdot \log_n 2)^{-1}$, for n = 2, 3, ... has $H(X) = +\infty$. *10*

3. (a) Prove that H(X, Y) = H(X) + H(Y/X). *3*

(b) Let (X, Y) have the following joint distribution :

Y \ X	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

The marginal distribution of X is $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8})$ and of Y is $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. Find H(X), H(Y), H(X/Y), H(Y/X) and H(X, Y). *7*

4. Let (X^n, Y^n) be the sequences of length 'n' drawn independent and identically distributed according to $P(X^n, Y^n) = \prod_{i=1}^{n} P(x_i, y_i)$, then prove that

(a) $Pr((X^n, Y^n) \in A_\varepsilon^{(n)}) \to 1$ as $n \to \infty$

(b) $\left[A_\varepsilon^{(n)}\right] \le 2^{n}\,(H(X, Y) + \varepsilon)$ 5+5

5. (a) Derive $I(X^n, Y^n) \le nC$ for all $P(X^n)$. Let Y^n be the result of passing X^n through a discrete memoryless channel.

(b) Prove $C_{FB} = C = \underset{P(X)}{\mathrm{Max}}\ I(X, Y)$, where C_{FB} is feedback capacity. 5+5

6. A (15, 11) Linear block code can be defined by the following parity array :

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(a) Show the Parity-Check Matrix for this code.

(b) List the Co-set leaders from the standard array.

(c) Is this code a perfect code ? Justify your answer.

(d) How many erasures can this code correct ? Explain. *10*

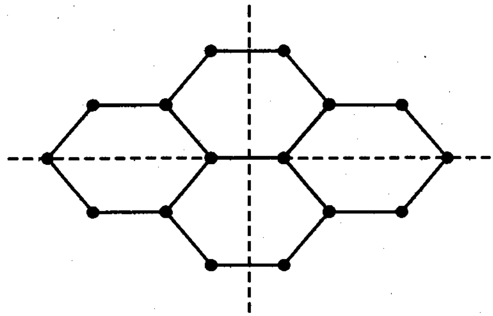
7. Draw the state diagram, tree diagram and trellis diagram for K = 3, rate $\frac{1}{2}$ code generated by $g_1(X) = 1 + X + X^2$, $g_2(X) = 1 + X^2$. *10*

8. Use the generator polynomial for (7, 3) R-S code to encode the message 010110111 in systematic form. Use polynomial division to find the parity polynomial and show the resulting code word in polynomial form and binary form. *10*

9. For a fixed error probability, show that the relationship between alphabet size M and required average power for MPSK versus QAM can be expressed as

$$\frac{\text{Average Power for MPSK}}{\text{Average Power for QAM}} \approx \frac{3M^2}{2(M-1)\,\pi^2}.$$ *10*

10. The figure given below shows several 16-array symbol constellations :



For the (5, 11) circular constellations, compute the minimum radial distances r_1 and r_2 if the minimum distance between each symbol must be 1 unit. *10*