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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

00322

December, 2016

ET-102 : MATHEMATICS – III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any* ***ten*** *questions. Use of scientific calculator is allowed.*

1. Show that the sequence $< x_n >$ given by

$$x_n = 1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \dots + \frac{1}{\lfloor n}, \; n \in N, \text{ is convergent.}$$ 7

2. Test the convergence of the series

$$\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots \text{ for all } p > 0.$$ 7

3. Find the Fourier series for the function $f(x) = |x|$ of period 2π. Also compute the values of the series at $x = 0$ and $x = -5\pi$. 7

4. Test the convergence of the series

$$\sum \frac{n^n x^n}{\lfloor n} \quad (x > 0).$$ 7

5. (a) Find the Inverse Laplace Transform of

$$\frac{s-1}{(s+3)(s^2+2s+2)}.$$

(b) If $\mathcal{L}$ represents Laplace Transform and if

$$\mathcal{L}\{F(t)\} = f(s), \text{ and } G(t) = \begin{cases} F(t-a), & t > a \\ 0, & t < a \end{cases},$$

then show that $\mathcal{L}[G(t)] = e^{-as} f(s)$. *4+3*

6. Using Laplace Transforms, solve the differential equation $\frac{d^2y}{dt^2} - t\frac{dy}{dt} + y = 1$,

given that $y(0) = 1$, $y'(0) = 2$. 7

7. If the population of a country doubles in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants ? 7

8. (a) Solve :

$$x \log x \frac{dy}{dx} + y = 2 \log x$$

(b) Find a particular integral of the differential equation $(D^4 + D^2 + 1)\, y = b\, e^{-x} \sin 2x$. *4+3*

9. Show that $x = 0$ is a singular point of the differential equation $x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = 0$.

Determine the indicial equation, its roots and recurrence formula. 7

10. Solve the partial differential equation

$$(x^2 - y^2 - z^2)\frac{\partial z}{\partial x} + 2xy\frac{\partial z}{\partial y} = 2xz.$$ 7

11. Show that the deflection u(x, t) satisfying the conditions

$$u_{tt} = u_{xx}, \quad 0 < x < \pi, \quad t > 0,$$
$$u(0, t) = 0 = u(\pi, 0), \quad t \geq 0$$
$$u(x, 0) = k \sin 2x, \quad 0 \leq x \leq \pi$$
$$\frac{\partial u}{\partial t}(x, 0) = 0, \quad 0 \leq x \leq \pi$$

is given by $u(x, t) = k \cos 2t \cos 2x$.

(Use the method of separation of variables). 7

12. (a) Determine the critical points of the transformation $w = f(z)$, where

$$f(z) = z^2 + 2z + 1.$$

(b) Find the bilinear transformation that maps ∞, i, 0 into the points 0, – i, ∞. 3+4

13. Determine the analytic function $w = u + i\,v$, if

$$u + v = e^{2x}\,[(x + y)\cos 2y + (x - y)\sin 2y]$$

and express w in terms of z. 7

14. Is the differential equation, whose characteristic equation is $s^5 - s^4 + 2s^3 + s^2 - 3s + 2 = 0$, stable under Hurwitz-Routh Criterion ? 7

15. (a) Expand cos z in a Taylor series about the point $z = \pi$.

(b) Find the Laurent series expansion of the function $f(z) = \dfrac{1}{z(1-z^2)}$ in the region $0 < |z| < 1$. *4+3*

16. Determine the residues at all singularities of the function $f(z) = \dfrac{z^3 e^{1/z}}{1-z^2}$. *7*

17. Evaluate

$$\int_0^{\pi} \frac{a\, d\theta}{a^2 + \cos^2\theta},$$

using the method of complex variables. *7*
