No. of Printed Pages : 5

**ET-101(A)**

**B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI**

**Term-End Examination**

00511

**December, 2016**

## ET-101(A) : MATHEMATICS – I

*Time : 3 hours* *Maximum Marks : 70*

***Note :*** *All the questions are **compulsory**. Use of scientific calculator is permitted.*

---

**1.** Answer any ***five*** of the following : *5×4=20*

(a) Differentiate $x^{\cos x}$ w.r.t. $(\cos x)^x$.

(b) If $y = \sin(m \sin^{-1} x)$, prove that

$$(1 - x^2) y_2 - x y_1 + m^2 y = 0.$$

Hence find $y_{n+2}$, using Leibnitz's theorem.

(c) If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(d) Find the ratio of the height to the radius of a closed cylinder of given volume and least surface area.

(e) Determine the values of 'p' and 'q' for which the function 'f' defined by

$$f(x) = \begin{cases} px^2 - 2x - q, & \text{when } x \geq 0 \\ \dfrac{px - 4}{\sqrt{x^4 + x^2 + 1}} + q\sqrt{1 - 3x}, & \text{when } x < 0 \end{cases}$$

is continuous at $x = 0$.

(f) Evaluate the following limits if they exist :

(i) $$\lim_{x \to 0} \frac{\sin x - \tan x}{ln(1 + x) - x + \frac{1}{2}x^2}$$

(ii) $$\lim_{x \to 3} \frac{\sqrt{x + 1} - \sqrt{x^2 - 5}}{\sin^{-1}(x - 3)}$$

2. Answer any ***four*** of the following : *4×4=16*

(a) Let $f : [0, 1] \to \mathbf{R}$ be a function defined by $f(x) = 2x$. Let $P_1 = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ and

$P_2 = \{0, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, 1\}$ be two partitions of the interval $[0, 1]$. Show that $L(P_2, f) \leq U(P_1, f)$.

(b) Solve the differential equation

$$xy - x^2 = y^2 \frac{dx}{dy}.$$

(c) Find the area between the cycloid;

$x = a(t - \sin t)$, $y = a(1 - \cos t)$ and its base.

(d) Evaluate the following integrals :

(i) $$\int_1^2 \frac{dx}{(x+1)\sqrt{x^2 - 1}}$$

(ii) $$\int_0^{\pi/4} \frac{1 + \sin 2x}{1 + \cos 2x} \cdot e^{2x} \, dx$$

(e) Evaluate the following integrals :

(i) $$\int_{-2}^{2} \{|x| + |x - 1|\} \, dx$$

(ii) $$\int_0^{\pi/4} \frac{\sec^2 x \, dx}{\sqrt{3 - 2\tan x - \tan^2 x}}$$

3. Answer any ***four*** of the following : *4×4=16*

(a) Find the directional derivative of $\phi = 2x^3 - 3yz$ at the point (2, 1, 3) in the direction parallel to a line, whose direction ratios are 2, 1, 2.

(b) Prove that

$$\nabla^2 \left( \frac{x}{r^2} \right) = - \frac{2x}{r^4},$$

where $r^2 = x^2 + y^2 + z^2$.

(c) Find curl grad $r^n$, where

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

(d) If $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$,

evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve,

$x = t, \ y = t^2, \ z = t^3$ from $t = 0$ to $t = 1$.

(e) Verify Green's theorem in a plane for

$$\oint_C [(x^2 - 2xy)\,dx + (x^2y + 3)\,dy],$$

where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

4. Answer any ***three*** of the following : *3×6=18*

(a) Solve the following system of linear equations using matrices :

$$x + 2y - 3z = -4$$
$$3x - y + z = 4$$
$$-4x + 2y + 3z = 9$$

(b) Prove that

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3.$$

(c) If $\mathbf{T} : \mathbf{R}^2 \to \mathbf{R}^3$ be a linear transformation defined by $T(x, y) = (x, x + y, y)$, find its range, null space, rank and nullity.

(d) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}.$$

---