No. of Printed Pages : 3

MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

00302

M.Sc. (MACS)

Term-End Examination

December, 2014

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Answer any **five** questions from questions no. **1** to **6**. Use of calculators is **not** allowed.*

---

1. (a) Define minimum weight and minimum distance of a code. 3

(b) List all codewords of the binary code C with parity-check matrix

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

and find the minimum distance and the minimum weight of C. 5

(c) Define binary Hamming code. 2

2. (a) Let C be the code with parity-check matrix

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Encode the message 0110 and write the corresponding code-word. *3*

(b) Let $a > 1$ be an integer. Prove that $(a^r - 1) \mid (a^m - 1)$ iff $r \mid m$. *4*

(c) Find a primitive element in the finite field $F_2[x]/< 1 + x + x^3 >$. *3*

3. (a) Let α be a primitive ninth root of unity in $\mathbf{F}_{64}$. Factor $x^9 - 1$ over $\mathbf{F}_2$ into a product of irreducible factors in terms of α. *4*

(b) Define a cyclic code and give an example. *2*

(c) Compute the syndrome decoding table for the code $C = \{0000, 1011, 0101, 1110\}$. *4*

4. (a) Let C be the [15, 7] narrow-sense binary BCH code of designed distance $\delta = 5$, which has defining set $T = \{1, 2, 3, 4, 6, 8, 9, 12\}$. Let α be a primitive 15^{th} root of unity where $\alpha^4 = 1 + \alpha$ and the generator polynomial of C is

$$g(x) = 1 + x^4 + x^6 + x^7 + x^8.$$

If $y(x) = x + x^4 + x^7 + x^8 + x^{11} + x^{12} + x^{13}$ is received, find the transmitted code-word. *7*

(b) Write MacWilliams equations and find the weight distribution of the code C whose generator matrix is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$ 3

5. (a) Let p be an odd prime and a be a positive integer. Prove that a splitting of p^a over $\mathbf{F}_2$ given by the multiplier μ_{-1} exists iff $ord_p(2)$ is odd. 5

(b) Let p be an odd prime. Prove that 2 is a square modulo p iff $p \equiv \pm 1 \pmod 8$. 3

(c) Find the Gray image of the code-words of the code C generated by 2

$$G = [1 \quad 2 \quad 3 \quad 1 \quad 2].$$

6. (a) State the Hensel's Lemma. 2

(b) Write the Two-Way APP Decoding Algorithm using an (n, k) binary convolutional code with received vector y. 8