No. of Printed Pages : 4

MMT-004

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00820

December, 2014

MMT-004 : REAL ANALYSIS

Time : 2 hours *Maximum Marks : 50*

(Weightage : 70%)

Note : *Question no. 1 is* ***compulsory****. Answer any* ***four*** *questions from questions no.* ***2*** *to* ***7****.*

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1. State, whether the following statements are *true* or *false*. Give reasons for your answer. *5×2=10*

   (a) If (X, d) is a metric space, then $\rho(x, y) = \max\{1, d(x, y)\}$ is a metric on X.

   (b) Intersection of a dense set and a nowhere dense set in a metric space is always a dense set.

   (c) The function $f(x, y) = (x + y, \cos|xy|)$ is continuously differentiable on $\mathbf{R}^2$.

(d) If E is a measurable set, then each translate $E + y$ is also measurable, for $y \in \mathbf{R}$.

(e) The system $(Rf)(t) = \int_{-\infty}^{t-1} f(\tau)\, d\tau$ is causal.

2. (a) Verify whether the metrics $d_1(x, y) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ and $d_2(x, y) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$ on $\mathbf{R}^2$ are equivalent or not. *4*

(b) Verify the chain rule for $f : \mathbf{R} \to \mathbf{R}^2$ defined by $f(t) = (t^2, t)$ and $g : \mathbf{R}^2 \to \mathbf{R}^3$ defined by $g(x_1, x_2) = (x_2 \sin x_1, x_1^2 + x_2^2, \cos x_2)$. *4*

(c) Find outer measure of the following sets : *2*

(i) $A = [-1, 1] \cup \{x \in \mathbf{R} : x^3 - 8 = 0\}$

(ii) $B = (0, 1] \cup (\frac{1}{2}, 2) \cup (\frac{3}{2}, 4)$

3. (a) Let $p_1 : \mathbf{R}^2 \to \mathbf{R}$ be the function such that $p_1(x, y) = x$. Show that p_1 is uniformly continuous on $\mathbf{R}^2$. *3*

(b) Find the regions where $f(x, y) = (e^x \cos y, e^x \sin y)$ is locally invertible. Is this function invertible as a function $f : \mathbf{R}^2 \to \mathbf{R}^2$. Justify your answer. *4*

(c) Show that x_A is measurable if and only if $A \subset \mathbf{R}$ is measurable. 3

4. (a) State Lebesgue Dominated Convergence Theorem. Use this to find

$$\lim_{n \to \infty} \int_0^1 f_n(x)\,dx \quad \text{where} \quad f_n(x) = \frac{nx}{1 + n^2x^2}.$$ 5

(b) Let $F : \mathbf{R}^4 \to \mathbf{R}^3$ be defined by

$F(x, y, z, w) = (2x^2y,\ xyz,\ x^2 + y^2 + 2zw^2)$

Find $F'(a)$ at $a = (1, 0, 1, 0)$. 5

5. (a) Show that the closure of a connected set in a metric space is always connected. 4

(b) Find the critical points of

$f(x, y, z) = x^4 + 2y^2 + 3z^2 - 2x^2 + 4y - 12z + 3$

and check whether they are extreme points. 4

(c) Show that the system $(Rf)(t) = t\ f(t)$ is time-varying system. 2

6. (a) Use the method of Lagrange's multiplier to find the point on the line of intersection of the planes $x - y = 2$ and $x - 2z = 4$ that is closest to the origin. 5

(b) Find the Fourier series for $f(x) = x^2, \ -\pi < x < \pi.$ 3

(c) Suppose $f \in L^1(\mathbf{R})$, $\alpha \in \mathbf{R}$. Prove that if $g(x) = f(x - \alpha)$ then $\hat{g}(w) = \hat{f}(w - \alpha)$. 2

7. (a) Let X be a metric space such that given any two points x, y $\in$ X there exists connected set A such that x, y $\in$ A. Show that X is connected. 3

(b) Suppose $f, g \in L^1(\mathbf{R})$ and $h = f * g$. Show that $\hat{h}(w) = \hat{f}(w) \cdot \hat{g}(w)$. 4

(c) Check whether the system $(Rf)(t) = a + f(t)$ is linear or not. 3
