No. of Printed Pages : 4 **BICE-027**

B.Tech. – VIEP – MECHANICAL ENGINEERING / B.Tech. IN CIVIL ENGINEERING (BTMEVI / BTCLEVI)

01035

Term-End Examination

December, 2014

BICE-027 : MATHEMATICS-III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any **ten** questions. All questions carry equal marks. Use of scientific calculator is permitted.*

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1. Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, \quad -\pi < x < \pi$$

Hence show that

$$\sum \frac{1}{n^2} = \frac{\pi^2}{6}.$$ 7

2. Expand $f(x) = x \sin x, \ 0 < x < 2\pi,$ in a Fourier series. 7

3. For a function f(x) defined by

$$f(x) = |x|, \ -\pi < x < \pi,$$

obtain a Fourier series.

Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + ... = \frac{\pi^2}{8}.$ 7

4. Obtain a half-range cosine series for

$$f(x) = kx \text{ for } 0 \le x \le \frac{l}{2},$$

$$f(x) = k(l - x) \text{ for } \frac{l}{2} \le x \le l.$$

Also deduce the sum of the series

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots .$$ 7

5. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

Hence evaluate $\displaystyle\int_0^\infty \frac{\sin x}{x}\, dx.$ 7

6. Obtain Fourier sine transform of

$$f(x) = \begin{cases} \sin x & 0 < x < a \\ 0 & x > a \end{cases}$$ 7

7. Solve :

$$(x^2 - yz)\, p + (y^2 - zx)\, q = z^2 - xy.$$ 7

8. Solve :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\, \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x.$$ 7

9. Obtain the solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$$

using the method of separation of variables. 7

10. Using the method of separation of variables,

solve $$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x, 0) = 6\,e^{-3x}$. 7

11. A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x, t)$. 7

12. Solve the differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation, subject to the following conditions : 7

(i) u is not infinite for $t \to \infty$,

(ii) $\frac{\partial u}{\partial x} = 0$, for $x = 0$, and $x = l$,

(iii) $u = lx - x^2$ for $t = 0$, between $x = 0$ and $x = l$.

13. Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \text{ for } 0 < x < \pi, 0 < y < \pi,$$

with conditions given :

$$u(0, y) = u(\pi, y) = u(x, \pi) = 0,$$

$$u(x, 0) = \sin^2 x.$$ 7

14. A semi-circular plate of radius 'a' has its circumference kept at temperature

$$u(a, \theta) = k\theta (\pi - \theta)$$

while the boundary diameter is kept at zero temperature. Find the steady-state temperature distribution $u(r, \theta)$ of the plate assuming the lateral surfaces of the plate to be insulated. 7

15. A transmission line 1000 km long is initially under steady-state conditions with potential 1300 volts at the sending end (x = 0) and 1200 volts at the receiving end (x = 1000 km). The terminal end of the line is suddenly grounded but the potential at the source is kept at 1300 volts. Assuming the inductance and leakage to be negligible, find the potential $v(x, t)$. 7
