B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) Term-End Examination

00615　　　December, 2014

ET-102 : MATHEMATICS – III

Time : 3 hours　　　　　　　　　*Maximum Marks : 70*

Note : *Attempt any* **ten** *questions. All questions carry equal marks. Use of calculator is allowed.*

1. Test for convergence the series

   $$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots \infty$$　　7

2. Expand

   $$f(x) = x^2, \quad -1 < x < 1$$

   in a Fourier series.　　7

3. Solve :

   $$\frac{d^2y}{dx^2} + y = 2 x \sin x$$　　7

4. Solve :

   $$\frac{dy}{dx} = \frac{y - x}{y + x}$$　　7

5. The population of bacteria in a culture grows at a rate proportional to the number of bacteria present at time t. After 3 hours, it is observed that 400 bacteria are present. After 10 hours, 2000 bacteria are present. What are the initial number of bacteria ? 7

6. Determine the poles of the function $f(z) = \dfrac{5z^2 - 4z + 3}{(z + 1)(z + 2)(z + 3)}$, and the residue at each pole. 7

7. Find the transformation which maps the points -1, 0, 1 of the z-plane on to i, ∞, 0 of the w-plane respectively. 7

8. Write the given number in the form a + ib :
$$\frac{(5 - 4i) - (3 + 7i)}{(4 + 2i) + (2 - 3i)}$$
7

9. Find the four fourth roots of z = 1 + i. 7

10. Show that the given function is analytic in an appropriate domain :
$$f(z) = 4x^2 + 5x - 4y^2 + 9 + i(8xy + 5y - 1)$$
7

11. Expand $f(z) = \dfrac{1}{(z - 1)^2(z - 3)}$ in a Laurent series valid for $0 < |z - 3| < 2$. 7

12. Find the Laplace transform of
$$f(t) = 4t^2 - 5\sin 3t.$$
7

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13. Find the inverse Laplace transform of

$$\left\{ \frac{s}{s^2 + 2s - 3} \right\}$$

7

14. Solve :

$$x^2 (y - z) p + y^2 (z - x) q = z^2 (x - y)$$

7

15. Apply the Hurwitz-Routh criterion to determine the stability of the system whose characteristic equations are given by

7

(i) $s^4 + 3s^3 + 5s^2 + 2s = 10,$

(ii) $s^5 + 2s^4 + 3s^3 + 10s^2 - 12s - 10 = 0.$

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