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MINI-019

M.Tech. IN ADVANCED INFORMATION TECHNOLOGY – NETWORKING AND TELECOMMUNICATION (MTECHTC)

00274

Term-End Examination

December, 2014

MINI-019 : STATISTICAL SIGNAL ANALYSIS

Time : 3 hours *Maximum Marks : 100*

Note :

(i) *Section I is **compulsory**.*

(ii) *In Section II, solve any **five** questions.*

(iii) *Assume suitable data wherever required.*

(iv) *Draw suitable sketches wherever required.*

(v) *Use of calculator is allowed.*

SECTION I

1. Answer the following short answer questions : *10×3=30*

(a) There are n persons in a room. What is the probability that at least two persons have the same birthday ?

(b) Consider the experiment of tossing a fair coin repeatedly and counting the number of tosses required until the first head appears. Find the sample space of the experiment.

(c) Define independence of two events A and B, in terms of their probabilities.

(d) Show that $P(AB/C) = P(A/BC)\,P(B/C)$

(e) Let X be a continuous random variable with the pdf

$$f_X(x) = \begin{cases} x/k & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of k.

(f) Describe the principle of operation of maximum likelihood estimator.

(g) The pdf of a uniformly distributed random variable in the interval (a, b) is given by

$$f_X(x) = \begin{cases} 1/b - a & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

Find $F_X(x)$.

(h) Consider an event in two parallel paths (P1 and P2) between two points A and B. P1 has two switches and P2 has one switch. Express the closed path event between the points A and B in terms of switches closed conditions.

(i) What are the conditions under which a strict-sense stationary process should process X(t) ?

(j) Let Y = a X + b , where a and b are constants. Show that E(Y) = a E(X) + b using the definition of expected value E() of a random variable.

SECTION II

*Answer any **five** questions from this section :*

2. (a) State the axioms of probability. *7*

(b) Consider the experiment of throwing two fair dice simultaneously. Let A be the event that the first dice is odd, B, be the event that the second dice is odd, and C be the event that the sum is odd.

(i) List the outcomes of the vents A, B and C and find P(A), P(B), P(C). *3*

(ii) Find $P(A \cap C)$, $P(B \cap C)$, $P(A \cap B)$ and show that events A, B and C are pair-wise independent. *4*

3. The joint pdf of a bivariate random variable (X, Y) is given by

$$f_{XY}(x, y) = \begin{cases} k(x + y) & 0 < x < 2, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Find the value of k. *5*

(b) Find marginal pdfs $f_x(x)$ and $f_y(y)$. *5*

(c) Are X and Y independent ? *4*

4. (a) Derive a two state Markov discrete process and explain how it is used in digital communications. *8*

(b) For the following transition matrix, find the steady state probabilities. 6

$$T = \begin{bmatrix} 0 \cdot 5 & 0 \cdot 15 & 0 \cdot 35 \\ 0 \cdot 2 & 0 \cdot 55 & 0 \cdot 25 \\ 0 \cdot 25 & 0 \cdot 3 & 0 \cdot 45 \end{bmatrix}$$

5. Given a continuous random variable X with mean μ_x, variance σ_x^2 and pdf $f_x(x)$, where $f_x(x) = 0$ for $x < 0$. For any $a > 0$,

(a) Show that $P(X \geq a) \leq \frac{\mu_x}{a}$

(Markov inequality) 7

(b) Show that $P(|X - \mu_x| \geq a) \leq \frac{\sigma_x^2}{a^2}$

(Chebyshev inequality). 7

6. Consider a random process X(t) defined by

$X(t) = Y \cos(\omega t) \quad t \geq 0$

where k is a constant.

(a) Find the mean E(X(t)). 4

(b) Find the auto correlation function $R_x(t, s)$ of X(t). 5

(c) Find the auto covariance function $C_x(t, s)$ of X(t). 5

7. Write short notes on the following :

(a) Markov Process 4

(b) Priority in queuing models 4

(c) Stochastic process and any one of its applications in systems. 6

8. Let $(X_1, X_2, \ldots X_n)$ be a random sample of a Poisson random variable X with unknown parameter λ. The pdf of X is given by

$$f_x(x) = \begin{cases} e^{-\lambda} \dfrac{\lambda^x}{x!} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the likelihood function $L(\lambda)$ for estimating λ. 5

(b) Find the log-likelihood function $ln\ L(\lambda)$. 5

(c) Find the maximum likelihood estimate λ_{ML} of λ. 4