No. of Printed Pages : 4

MMTE-005

00190

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

Term-End Examination

December, 2012

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage 50%)

Note : *Do **any five** questions from question **1** to **6**. Use of calculators is **not** allowed.*

1. (a) Define linear and non-linear codes. Give one example for each. **4**

(b) Which of the following codes given by their generator matrices, are perfect ? **6**

(i) $G_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$.

(ii) $G_2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$.

Justify your answer.

2. (a) Find all possible code words of the code C whose generator matrix G is given as. 6

$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Find the Hamming weight of each code word and the minimum Hamming distance of the code. How many errors can it detect and how many can it correct ? Justify your answer.

(b) Compute the gcd $(x^4 + x^3 + x + 1, x^3 + 1)$ in the ring $\mathbf{F}_2[X]$ using Euclidean algorithm. 4

3. (a) Let $g(x) = 1 + x + x^3$ be the generator polynomial of a [7, 4] cyclic code. Construct the generator matrix and parity check matrix of the code. 5

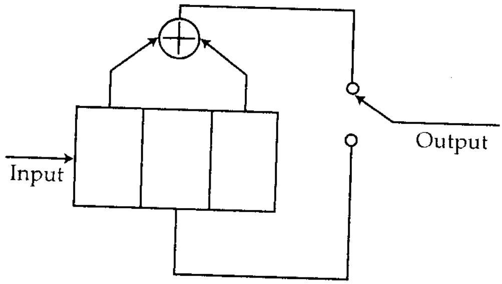
(b) (i) How many pairs of even like duadic codes of length 11 are there over $\mathbf{F}_3$? Justify your answer. 5

(ii) Construct their idempotents.

(iii) Construct the codes.

4. (a) Let C be the $\mathbf{Z}_4$-linear code of length 3 with generator matrix $G = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$. List the Gray image of the code C. 4

(b) 6



(i) Obtain the Trellis diagram of the convolutional encoder in the above figure.

(ii) Obtain the output of the encoder if the input message is 11011.

(iii) Decode the obtained coded output in (ii) using Viterli algorithm.

5. (a) Construct 3 different BCH codes over F_3 of length 13. 6

(b) Construct the Tanner graph for the [10, 6, 2] binary code with parity check matrix. 4

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

6. (a) Check whether $x^2 + x + 2$ is irreducible. Over $\mathbf{F}_3$. If $\alpha = x + <x^2 + x + 2>$, find the order of α in $\dfrac{F_3\,[x]}{<x^2+x+2>}$. 4

(b) Compute a table of Stirling numbers S (r, v) of the second kind for $1 \leq r \leq 0 \leq 6$. 4

(c) Check that the code over $\mathbf{Z}_4$ with the following generator matrix is self dual. 2

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 3 & 1 & 2 & 1 \\ 0 & 1 & 0 & 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 & 3 & 3 & 3 & 2 \\ 0 & 0 & 0 & 1 & 2 & 3 & 1 & 1 \end{bmatrix}.$$