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MMT-005

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2012

MMT-005 : COMPLEX ANALYSIS

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. **1** is **compulsory**. Attempt **any three** questions from question number **2** to **5**. Use of calculator is **not** allowed.*

1. State giving reasons whether the following statements are true or false : **5x2=10**

(a) The complex function $f(z) = \dfrac{\bar{z}}{|z|^2}$ is analytic every where.

(b) The linear fractional transformation from $0, 1, \infty$ to $\infty, -1, 1$ is $\dfrac{z-2}{z}$

(c) If $f(z) = \dfrac{5z+7}{z^2+2z-3}$, then $\oint_C f(z) = 0$ where c is the circle $|z-2| = 2$.

(d) The radius of convergence of power series

$$\sum_{n=0}^{\infty} \cos\left(\frac{n\pi}{3}\right) z^n \text{ is 1.}$$

(e) $e^{\frac{1}{z-1}}$ has simple poles at $z = 1$.

2. (a) Find the harmonic conjugate $v(x, y)$ of the harmonic function $u(x,y) = xy + x + 2y - 5$ and also an analytic function $f(z) = u + iv$ if $f(2i) = -1 + 5i$. 3

(b) If $f(z) = 1 - z$ for $|z| \leq 1$ then show that $|f(z)|$ attains its maximum value when $z = -1$. 2

3. (a) If $f(a) = \int_{|z|=5} \frac{2z^2 + 5z - 9}{(z - a)^2} dz$ for $|a| \neq 5$, determine $f(a)$ and also find $f'(1 + i)$ and $f'(1 - i)$. 3

(b) Find the Laurent series expansion of $f(z) = \frac{1}{z(1 - z)}$ in the region $0 < |z - 1| < 1$ in the powers of $(z - 1)$ and hence find 2

$$\oint_C f(z)\, dz$$

where c : $|z - 1| = \frac{1}{2}$.

4. (a) State Liouville's theorem. Show that if f is an entire function such that $f(z) = 100e^{Rez}$, then $f(z) = ce^z$ for some constant c. 3

(b) Show that the mapping $w = \frac{1}{z}$ transforms the line $x - 2y + 1 = 0$ into the circle whose center is $\left(\frac{-1}{2}, -1\right)$ and radius is $\frac{\sqrt{5}}{2}$. 2

5. Evaluate $\int_0^{2\pi} \frac{dx}{(2 + \cos x)^2}$ 5