MMT-002

01820

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2012

MMT-002 : LINEAR ALGEBRA

Time : 1½ hours *Maximum Marks : 25*

(Weightage 70%)

Note : *Question No. 5 is **compulsory**. Answer **any three** questions from question Nos. **1** to **4**. Use of calculators is **not allowed**.*

1\. (a) Let $T\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2a+2b+c \\ 2a+3b+c \\ 2b+c \end{bmatrix}$ be a linear operator on $\mathbf{R}^3$. Find the matrix of T with respect to the basis $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. **3**

If B_0 is the standard basis of $\mathbf{R}^3$ find an invertible matrix P such that $[T]_{B_0} = P\,[T]_B\,P^{-1}$.

(b) Find the least square solution for the system 2

$x + y = 1$

$x - y = 0$

$4y = 3$

2. (a) Write the Jordan Canonical Form for the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 2 \\ 0 & -1 & 3 \end{bmatrix}$. 3

(b) Let $A = \begin{bmatrix} 1 & i \\ -i & 1+2i \end{bmatrix}$. Find a unitary matrix U such that U*AU is upper triangular. 2

3. (a) Let $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$. Show that A is a positive definite matrix. Also find a positive definite matrix B such that $A = B^2$. 3

(b) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ find e^A. 2

4. Find the SVD of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$. 5

5. Which of the following statements are true and which are false ? Give reasons for your answers. **10**

(a) Two similar matrices have the same minimal polynomial.

(b) If the matrix of a predator prey system is

$$\begin{bmatrix} 0.3 & 0.1 \\ -0.1 & 2 \end{bmatrix},$$ both the predator and prey populations perish with time.

(c) If D is a diagonal matrix and N is a nilpotent matrix then N and D commute.

(d) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, then AB is positive definite.

(e) If B is the Moore-Penrose inverse of A, then $(AB)^2 = AB$.
