ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

December, 2012

ET-102 : MATHEMATICS - III

Time : 3 hours   *Maximum Marks : 70*

*Note :   Answer any **ten** questions. Use of calculator is **allowed**.*

1. Show that the sequence $<a_n>$ defined by :   7

    $$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

    does not converge.

2. Test for convergence the series :   7

    $$\frac{1}{3}x + \frac{1}{3}, \frac{2}{5}x^2 + \frac{1}{3}, \frac{2}{5}, \frac{3}{7}x^3 + - - + \frac{1.2. - - n}{3.5. - - (2n+1)}x^n + ..$$

    for all positive valves of x.

3. Obtain Fourier series for the function $f(x)$ given   7
    by :

    $$f(x) = \begin{cases} |x| & \text{for} & -\frac{3}{2} \le x < -1 \\ 0 & \text{for} & -1 \le x \le 1 \\ 0 & \text{for} & 1 \le x \le \frac{3}{2} \end{cases}$$

4. Show that the series $\qquad$ 7

$$\frac{\log 2}{2^2} - \frac{\log 3}{3^2} + \frac{\log 4}{4^2} - \text{-- -- --}$$

converges.

5. (a) Evaluate $L\ (e^{-2t} \cos 4t)$, where L denotes $\quad$ 3+4 Laplace Transform.

(b) Using gravitation Theorem, evaluate :

$$L^{-1} \left\{ \frac{1}{(s+1)\ (s^2+1)} \right\}$$

6. Solve $y'' + 9y = x$, given that $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$, $\quad$ 7

using laplace transform method.

7. Find the characteristic function, transfer function, $\quad$ 7 frequency response function and characteristic roots of the differential equation $(D^2 + 2D + 1)y = e^{5x}$ and then find its particular solution.

8. Apply the Husuitz-Routh criterion to determine $\quad$ 7 the stability of the system, whose characteristic equation is given by :

$s^5 - 3s^4 + 2s^3 + 6s^2 - 13s - 11 = 0$.

9. In a certain isolated population p(t), the rate of   7
    population growth $\dfrac{dp}{dt}$ is equal to $p + \dfrac{K}{\epsilon}p^2$,
    where K and ϵ are both positive constants. If
    $p(0) = 1$, then find the limiting population as
    $t \to \infty$.

10. (a) Solve $y \log y \dfrac{dx}{dy} + x - \log y = 0$.   4+3

    (b) Find a particular integral of the differential
         equation :
         $$(D^4 - 4D^3 + 6D - 7)y = e^{2x}.$$

11. Shown that $x = 0$ is a regular singular point of the   7
    differential equation :
    $$xy'' + 2xy' + 6y = 0$$
    Determine the indicial equation, its roots and
    recurrence formula.

12. Solve the partial differential equation   7
    $$x^2p + y^2q + z^2 = 0.$$

13. Using the method of separation of variables, solve   7
    IBVP :
    $$\frac{\partial^2 u}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 u}{\partial t^2}, \quad 0 < x < e, \, t > 0.$$
    with $u(0, t) = 0$, $t \geq 0$; $u_x(e, t) = 0$
    $u(x, 0) = f(x)$ for $0 \leq x \leq e$; $u_t(x, 0) = 0$.

14. Determine the analytic function $w = u + iv$ if $\quad$ 7
$u = e^{2x} (x \cos 2y - y \sin 2y)$.

15. Find the bilinear mapping that maps the points $\quad$ 7
$z_1 = \infty$, $z_2 = -i$ and $z_3 = 1$ into the points $w_1 = 0$,
$w_2 = i$ and $w_3 = \infty$.

16. Using complex $\dfrac{x}{\pi}$ Variable Method, evaluate : $\quad$ 7

$$\int_0^\pi \frac{2 + 3 \cos \theta}{6 + 5 \cos \theta} \, d\theta \, .$$

17. Find the value of the integral $\quad$ 7

$$\int_C \frac{3z - 7}{(z - 1)^2 \, (3z + 7)} \, dz \, ,$$

where C is the circle $|z| = 2$.

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