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ET-101(A)

B.Tech. Civil (Construction Management)/ B.Tech. Civil (Water Resources Engineering) B.Tech. (Aerospace Engineering) BTCLEVI/BIMEVI/BTELVI/BTECVI/BTCSVI

Term-End Examination

December, 2012

01893

ET-101(A) : MATHEMATICS-I

Time : 3 hours *Maximum Marks :* **70**

Note : *All questions are* ***compulsory****. Use of calculator is allowed.*

1. Answer *any five* of the following : **5x4=20**

(a) If $f(x)$ be a function of real variable x, $f(x)$ defined by :

$$f(x) = \begin{cases} -x & \text{when } x \leq 0 \\ x & \text{when } 0 < x < 1 \\ 2-x & \text{when } x \geq 1 \end{cases}$$

Show that $f(x)$ is continuous at $x=0$, and also at $x=1$.

(b) If $y=\sin(m \sin^{-1}x)$, prove that $(1-x^2)y_{n+2}-(2n+1)x.y_{n+1}+(m^2-n^2)y_{n=0}$.

(c) Evaluate *any one* of the following :

(i) $\lim\limits_{x \to 0} \dfrac{1-\cos x}{x\sin x}$

(ii) $\lim\limits_{x \to \frac{\pi}{2}} (1+\cos x)^{2\sec x}$

(d) Find the angle between the curves $y^2 = 4ax$ and $x^2 = 4ay$ at their point of intersection other than origin.

(e) Show that the semi-vertical angle of the cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$.

(f) Find value of Jacobian $\frac{\partial(u,v)}{\partial(\gamma,\theta)}$ where

$u = x^2 - y^2$, $v = 2xy$ and $x = \gamma\cos\theta$, $y = \gamma\sin\theta$

2. Answer *any four* of the following : **4x4=16**

(a) Evaluate any of the following :

(i) $\int \operatorname{cosec} x \, dx$

(ii) $\int \sin^5 x \cos^4 x \, dx$

(b) Show that : $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}$

(c) Find the area lying between the porabola $y = 4x - x^2$, and the line $y = x$.

(d) The velocity of a train which starts from rest is given by the following table, the time being reckoned in minutes from the restart and speed in kilometer per hour.

Time (in minutes)	2	4	6	8	10	12	14	16	18	20
Speed (Km/hr)	10	18	25	29	32	20	11	5	2	0

Estimate approximately by Simpson's rule, the total distance run in 20 minutes.

(e) Solve (any one of the following)

(i) $(4x + y)^2 \dfrac{dx}{dy} = 1$

(ii) $\tan y \dfrac{dy}{dx} + \tan x = \cos y \cos^2 x$

(f) Prove that $y = f(x + at) + g(x - at)$ satisfies :

$$\frac{\partial^2 x}{\partial t^2} = a^2 \left(\frac{\partial^2 y}{\partial x^2} \right)$$

Where f and g are assumed to be at least twice differentiable and a is any constant.

3. Answer *any four* of the following : **4x4=16**

(a) Verify whether the three vectors $\vec{\alpha} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{\beta} = \hat{i} + \hat{j} - 3\hat{k}$ and $\vec{\gamma} = 7\hat{i} - 4\hat{j} + \hat{k}$ are at right angles to each other or.

(b) Find the constants a, b, c so that the vector :

$$W = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

becomes irrotational.

(c) Prove that curl $\left(\vec{\gamma} \times \vec{C}\right) = -2\vec{C}$, where

$\vec{\gamma} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{C}$ = a constant

vector $C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$.

(d) Verify : Curl $\vec{B} = O$ if ,

$\vec{B} = (2x - 2y)\hat{i} + \left(-2x + 2y + z^2\right)\hat{j} + 2yz\hat{k}$,

then find ϕ such that $\vec{B}$ = grade ϕ.

(e) If $F = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, then find $\nabla.F$ and $\nabla \times F$.

(f) Given :

$a = 4\hat{i} + 5\hat{j} - \hat{k}, b = \hat{i} - 4\hat{j} + 5\hat{k}, C = 2\hat{i} + \hat{j} - \hat{k}.$

Find a vector μ which is perpendicualr to both a and b and which satisfy $\mu.c. = 21$.

4. Answer *any six* of the following : **6x3=18**

(a) Verify Cayley-Hamilton theorem for the matrix :

$$B = \begin{bmatrix} 1 & 2 \\ 8 & 7 \end{bmatrix} \text{ and hence compute } B^{-1}.$$

(b) Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Prove that $A^{-1} = A^T$.

(c) Express the following matrix as the sum of a symmetric and a skew-symmetric matrix :

$$A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$$

(d) Show that : $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is an orthogonal matrix.

(e) Matrices A and B are such that :

$$3A - 2B = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix} \text{ and}$$

$$-4A + B = \begin{bmatrix} -1 & 2 \\ -4 & 4 \end{bmatrix}. \text{ Find A and B.}$$

(f) Determine the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(g) Find the rank of the matrix :

$$A = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$$

(h) Solve the following equations by matrix method :

$$3x + y + 2z = 3$$

$$2x + 3y - z = -3$$

$$x + 2y + z = 4$$
