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MMT-009

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

Term-End Examination

December, 2013

MMT-009 : MATHEMATICAL MODELLING

Time : 1½ hours | *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any five questions. Use of calculator is **not** allowed.*

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1. (a) In a tumour region, the control parameters of growth and decay of a tumour are respectively 70 and 30 per month. Emigration occurs at a constant rate of 2×10^3 cells per month. Use these assumptions to formulate the logistic model of the tumour size. Solve the formulated equation and describe the long term behaviour of the tumour size when the initial size of the tumour is 4×10^6 cells. **3**

   (b) Indifference curves of an investor cannot intersect. Is this statement true ? Give reasons for your answer. **2**

2. Consider the discrete time population model given by : 5

$$N_{t+1} = \frac{rN_t}{1+\left(\frac{N_t}{K}\right)^b}, \text{ for a population } N_t,$$

where K is the carrying capacity of the population, r is the intrinsic growth rate and b is a positive parameter. Determine the non-negative steady state and discuss the linear stability of the model for $0 < r < 1$. Also find the first bifurcation value of the parameter r.

3. (a) Compare the risk of two securities 1 and 2 whose return distributions are given below : 3

Possible rates of returns for security		Associated Probability
1	2	$P_1j = P_2j$
0.19	0.09	0.13
0.17	0.16	0.15
0.11	0.18	0.42
0.10	0.11	0.30

(b) A simple model including the seasonal change that affects the growth rate of a population is given by $\frac{dx}{dt} = C\, x(t) \sin t$, where C is a constant. If x_0 is the initial population then solve the equation and determine the maximum and minimum populations. 2

4. Do the stability analysis of any one of the equilibrium solutions of the following competing species system of equations with diffusion and advection : 5

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2} - V_1 \frac{\partial N_1}{\partial x}$$

$$\frac{\partial N_2}{\partial t} = - d_1 N_2 + C_1 N_1 N_2 + D_1 \frac{\partial^2 N_2}{\partial x^2} - V_2 \frac{\partial N_2}{\partial x} ,$$

$0 \le x \le 2$

Where V_1 and V_2 are constant advection velocities in x direction of the two populations with densities N_1 and N_2, respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate, C_1 is the conversion rate. D_1 and D_2 are diffusion constants. The initial and boundary conditions are :

$N_i (x, 0) = f_i (x) > 0, 0 \le x \le L, i = 1, 2$

$N_i = \overline{N_i}$ at $x = 0$ and $x = L \forall t, i = 1, 2$

Where $\overline{N_i}$ are the equilibrium solutions of the given system of equations. Also write the limitations of the model.

5. A company has factories at F_1, F_2 and F_3 which supply to warehouses at W_1, W_2 and W_3. Weekly factory capacities are 200, 160 and 90 units respectively. Weekly warehouse requirements are 180, 120 and 150 units respectively. Unit shipping costs (in rupees) are as follows : 5

	W_1	W_2	W_3	Supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	

Determine the optimal distribution for this company to minimize total shipping cost.

6. (a) Find a linear demand curve that best fit the following data : **3**

x	1	2	3	4	5
y	14	27	40	55	68

(b) Find the number of covariances needed for an evaluation of 200 securities using the Markowitz model. Also calculate the total number of pieces of information needed. **2**