**MMT-003**

00143

# M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

## Term-End Examination

## December, 2013

## MMT-003 : (ALGEBRA)

*Time : 2 hours* *Maximum Marks : 50*

*Weightage 70%*

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*Note : Question no. 1 is **compulsory**. Do **any four** questions from questions no. 2 to 6. Use of Calculators are **not** allowed.*

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**1.** State which of the following statements are true and which are false. Give reasons for your answer. **5x2=10**

(a) If $m > 1$ and $n > 1$ are natural numbers with $m > n$, there is a group G of order m and a set with n elements such that G operates transitively on S.

(b) There is a group of order 14 in which all the elements have order 7.

(c) It is not possible for a group of order 36 to have an irreducible representation of dimension 9.

(d) There exists $3 \times 3$ orthogonal matrix with $\left(\frac{1}{4}, \frac{-1}{2}, \frac{3}{4}\right)$ as its first row.

(e) The eigen values of $\begin{pmatrix} 1 & \pi \\ 0 & i \end{pmatrix}$ are algebraic over Q.

2. (a) For $n \geq 3$, show that the symmetric group Sn is not cyclic, but can be generated by 2 elements. 4

(b) Solve the set of congruences 3

$$2x \equiv 1 \pmod 5$$
$$x \equiv 3 \pmod 4$$

(c) Let S be a non empty set. Show that Map (S, S), the set of all mappings from S to S is a monoid. Determine the group kernel of Map(S, S). 3

3. (a) Evaluate the legendre symbol $\left(\frac{13}{997}\right)$. 2

(b) Determine all the irreducible representations of $D_3$. Further, write down the character table of $D_3$. 6

(c) Find the invariant factors of $Z_8 \times Z_{12} \times Z_{15}$. 2

4. (a) Show that $L=\{x^n y \mid n \geq 0\}$ is a regular language. 3

(b) Check if the ISBN number 978-81-266-4945-7 is a valid ISBN number. 2

(c) Let $\alpha$, $\beta$ be complex numbers. Prove that if $\alpha+\beta$ and $\alpha\beta$ are algebraic numbers, then $\alpha$ and $\beta$ are also algebraic 5

5. (a) Let $F$ be a finite field. Show that the product of all the non-zero elements of $F$ is $-1$. 3

(b) The matrix $A = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ has order 3 and therefore it defines a matrix representation of the cyclic group G of order 3. Find a G-invariant, positive definite hermitian form on $C^n$. 4

(c) Find the Stabiliser of $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ under conjugation in $GL_2(\mathbf{R})$. 3

6. (a) Prove that $SP_2(\mathbf{R}) = SL_2 = (\mathbf{R})$ but that $SP_4(R) \neq SL_4(R)$. 5

(b) Let $K=F(\alpha)$ where $\alpha$ is a root of the irreducible polynomial $f(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_0$, where $n \geq 2$ . Find $\alpha^{-1}$ and $\alpha^{-2}$ explicitly in terms of $\alpha$ and the coefficients $a_i$. 3

(c) Check whether $F_7(\sqrt{3})$ and $F_7(\sqrt{5})$ are isomorphic as vector spaces. 2

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