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**ET-101(A)**

# B.Tech. Civil (Construction Management)/ B.Tech. Civil (Water Resources Engineering) B.Tech. (Aerospace Engineering) BTCLEVI/BIMEVI/BTELVI/BTECVI/BTCSVI

**Term-End Examination**

**December, 2013**

01700

## ET-101(A) : MATHEMATICS-I

*Time : 3 hours* *Maximum Marks : 70*

***Note :*** *All questions are* ***compulsory.***
*Use of scientific calculator is* ***permitted.***

**1.** Answer **any five** of the following : **5x4=20**

(a) Examine the differentiability of the function.

$$f(x) = \begin{cases} x, & -\infty < x < 0 \\ 1, & 0 \le x < 2 \\ 3 - x & 2 \le x \end{cases}$$

Also draw its graph.

(b) Find $\dfrac{d^2 y}{dx^2}$ at $t = \pi/4$ for the function

$x = a$ cost, $y = a$ sint.

(c) Expand the polynomial $x^5 - 3x^4 + 2x^3 - x^2 + 1$ in powers of $(x-2)$ using Taylor's Theorem.

(d) Find the angles of intersection of $xy = 10$ and $x^2 + y^2 = 29$.

(e) Find the maximum and minimum of $f(x) = \sin x\ (1 + \cos x)$.

(f) Show that $\lim_{x \to \infty} (x^2 + 1)^{1/ln^x} = e^2$.

(g) Use the differential to estimate $\sqrt{27}\ \sqrt[3]{1021}$

(h) If $u = f\left(\frac{x}{z}, \frac{y}{z}\right)$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial v}{\partial z} = 0$$

(i) If $u = x + y + z,\ y + z = uv,\ z = uvw$ find

$$\frac{\partial(x, y, z)}{\partial(u, v, w)}$$

2. Answer **any four** of the following : **4x4=16**

(a) Find the area enclosed by the cardiod $r = a\ (1 - \cos\theta)$.

(b) Find the area of the astroid $x = a\cos^3 t$, $y = b\sin^3 t$, $0 \le t \le 2\pi$.

(c) Use the trapezoidal rule with six subdivisions to evaluate

$$\int_0^{\pi} \sqrt{\cos x}\, dx$$

(d) Find the perimeter of the loop of the curve $r = a\ (\theta^2 - 1)$.

(e) Find the volume of the right circular cone of height 5 and radius of the circular base 2.

(f) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about the line $y = 0$.

3. Answer **any four** of the following : **4x4=16**

(a) Find the directional derivative of $x^2 + y^2 + 4xyz$ at $(1, -2, 2)$ in the direction of $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$.

(b) Define curl of a vector function. Give its physical interpretation. If $\vec{F} = \nabla\ (x^3 + y^3 + z^3 - 3xyz)$, then what is $\nabla \vec{F}$ ?

(c) Evaluate $\int_c (x^2 + y^2 + z^2)\, ds$, where c is the arc of circular helix $\vec{r}\ (t) = \cos t\ \hat{i} + \sin t\ \hat{j} + 3t\hat{k}$.

(d) Evaluate $\oint[(x^2 - 2xy)\ dx + (x^2y + 3)dy]$ around the boundary C of the region $y^2 = 8x$, $x = 2$.

(e) State Stoke's Theorem. Derive Green's Theorem in the plane as a special case of Stoke's Theorem.

(f) Verify divergence theorem for the sphere

$x^2 + y^2 + z^2 = a^2$ if $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$.

**4.** Answer **any six** of the following. **6x3=18**

(a) Find an orthogonal basis of $R^4$ containing $(1, -2, 1, 3)$ and $(2, 1, -3, 1)$.

(b) Check whether the transformation $T(x, y, z) = (x, y, 4z)$ is invertible ? If so, find $T^{-1}$.

(c) Define rank of a matrix. Let A be a $3 \times 4$ matrix such that,

$$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

What can you say about the rank of A ?

(d) Determine the values of a for which the following system of linear equations has

(i) a unique solution

(ii) infinite number of solutions.

$x + y + z = 2,$ $\quad x + 2y + z = -2,$

$x + y + (a - 5)z = a - 4.$

(e) Define eigen value problem of a matrix.

Find the eigen values of the matrix $\begin{bmatrix} 1 & i \\ 1 & 1 \end{bmatrix}$

(f) Show that a matrix is singular if and only if one of its eigen values is zero.

(g) Verify Caley - Hamilton Theorem in case of

the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$

(h) Compute $e^A$ where $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$.