BMTC-131

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(BSCG/BAG)

CALCULUS

Valid from 1st July, 2019 to 30th June, 2020

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2019-20)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, A, B and C. The maximum marks of all the parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

……

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COURSE CODE: ……..

COURSE TITLE: ……..

ASSIGNMENT NO.: ……..…

STUDY CENTRE: ……..….. **DATE:** ……....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto **June, 2020**. If you have failed in this assignment or fail to submit it by June, 2020, then you need to get the assignment for the year 2020-2021 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: BMTC-131
Assignment Code: BMTC-131/TMA/2019-20
Maximum Marks: 100

PART – A (35 marks)
(Based on Blocks 1 and 2 of the course)

1. Which of the following statements are true, and which are false? Give reasons for your answers in the form of a short proof or a counterexample.

   i) There are at least two ways of describing the set $\{7,8,...\}$.

   ii) Any function with domain $\mathbb{R}\times\mathbb{R}$ is a binary operation.

   iii) The graph of every function from $[0,1]$ to $\mathbb{R}$ is infinite.

   iv) The function $f:\mathbb{R}\to\mathbb{R}$, defined by $f(x)=x|x|$, is an odd function.

   v) The domain of the function $f\circ g$, where $f(x)=\sqrt{x}$ and $g(x)=\sqrt{2-x}$, is $]-\infty,2]$. (10)

2. a) Obtain the 6^{th} roots of (-7), and represent them in an Argand diagram. (6)

   b) Using the $\varepsilon-\delta$ definition, show that $\lim_{x\to 2}(3x-5)=1$. (4)

3. a) Find the nature of the roots of the polynomial $2x^5+x^3+5x+1$. (3)

   b) Let $f(x)=\begin{cases}\sqrt{-x} & \text{, if } x<0\\ 3-x & \text{, if } 0\le x<3\\ (x-3)^2 & \text{, if } x>3\end{cases}$

      i) Check whether f is discontinuous. If yes, find where?

      ii) Give a rough sketch of the graph of f. (5)

   c) Express the function $g:\mathbb{R}^+\to\mathbb{R}^+$, defined by $g(x)=\frac{1}{\sqrt{x+\sqrt{x}}}$, as a composition of three functions. (2)

4. a) Can the Intermediate Value Theorem be applied to show that there is a root of the equation $x^5-x^3+3x-5=0$ in the given interval $]1,2[$? If yes, apply it. (3)

   b) Find an expression for the function whose graph consists of the line segment from the point $(-2,2)$ to the point $(-1,0)$ together with the top half of the circle with centre at the origin and radius 1. (2)

PART – B (40 Marks)
(Based on Blocks 3 and 4 of the course)

5. a) The equation of motion of a particle is $s = t^3 - 3t$, where s is in metres and t is in seconds. Find

   i) the velocity and acceleration as functions of t,

   ii) the acceleration after 2 seconds,

   iii) the acceleration, when the velocity is 0. (3)

   b) For which values of a and b is the following equation true?

   $$\lim_{x\to 0}\left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2}\right) = 0 .$$ (3)

   c) Find two positive integers such that the sum of the first number and four times the second number is 1000, and the product of the numbers is as large as possible. (4)

6. a) For which values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ when $x = 2$? (2)

   b) Differentiate y w.r.t. x in the following cases:

   i) $y = \sin(\sin x)$

   ii) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

   iii) $y = e^{\cos x} + \cos(e^x)$

   iv) $y = \ln(x \ln x)$ (8)

7. Which of the following statements are true? Give reasons for your answers, in the form of a short proof or a counterexample.

   i) $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$

   ii) The inverse function of $y = e^{3x}$ is $y = \dfrac{1}{3}\ln x$.

   iii) If f is increasing and $f(x) > 0$ on an interval I, then $g(x) = \dfrac{1}{f(x)}$ is decreasing on I.

   iv) An equation of the tangent line to the parabola $y = x^2$ at $(-2,4)$ is $y - 4 = 2x(x+2)$.

   v) If f is one-one onto and differentiable on $\mathbb{R}$, then $(f^{-1})'(6) = \dfrac{1}{f'(6)}$. (10)

8. Trace the curve $y = \sqrt[3]{x^2 - 1}$, and state all the properties you use to trace it. (10)

PART – C (25 Marks)
(Based on Block 5 of the course)

9. Integrate the following functions w.r.t. x:

i) $\dfrac{1}{(2x+1)^{3/2}}$ ii) $\sin(2x+3)$

iii) $\operatorname{cosec}(4x)$ iv) $\dfrac{1}{\sqrt{1-9x^2}}$

v) $\dfrac{1}{1+4x^2}$ (5)

10. a) Integrate the following functions w.r.t. x using substitution:

i) $\int_0^1 x^3\sqrt{x^4+1}\,dx$ ii) $\int_0^{\pi/4} \tan^7 x \sec^2 x\,dx$

iii) $\int \sec^2(\cos x)\sin x\,dx$ (6)

b) Derive the reduction formula

$$\int\left(x^2+a^2\right)^{\frac{n}{2}}dx = \frac{x\left(x^2+a^2\right)^{\frac{n}{2}}}{n+1} + \frac{na^2}{n+1}\int\left(x^2+a^2\right)^{\frac{n}{2}-1}dx.$$

Use the formula to integrate $\int\left(x^2+a^2\right)^{\frac{5}{2}}$.

(**Hint:** Integrating by parts,
$\int\left(x^2+a^2\right)^{\frac{n}{2}}dx = x\left(x^2+a^2\right)^{\frac{n}{2}} - n\int\left(x^2+a^2\right)\left(x^2+a^2\right)^{\frac{n}{2}-1}dx + na^2\int\left(x^2+a^2\right)^{\frac{n}{2}-1}dx$) (4)

11. a) Use integration by parts to integrate $\int x\sec x\tan x\,dx$ (2)

b) Integrate $\int(3x+1)\sqrt{4x^2+12x+5}\;dx$ (3)

c) Integrate $\int\dfrac{x^2+x+5}{(x^2+4)(x+1)}dx$ (2)

d) Find the length of the portion of the parabola $x=3t^2, y=6t$ cut off by the line $3x+y-3=0$. (3)