BMTC-132

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

(BSCG / BAG)
DIFFERENTIAL EQUATIONS

Valid from 1st Jan, 2020 to 31st Dec, 2020

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for B.Sc. that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, Part A, Part B, Part C, covering all the blocks of the course. The total marks of the three parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: …………………………..….. **DATE:** ……………………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) Solve Part A, Part B and Part C of this assignment, and **submit the complete assignment answer sheets within the due date.**

6) The assignment answer sheets are to be submitted to your Study Centre within the due date. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is **valid from 1st Jan, 2020 to 31st Dec, 2020**. If you have failed in this assignment or fail to submit it by Dec, 2020, then you need to get the assignment for the year 2021, and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the examination form for this course** until you have submitted this assignment.

We wish you good luck.

ASSIGNMENT

Course Code: BMTC-132
Assignment Code: BMTC-132/TMA/2020
Maximum Marks: 100

Part A (30 Marks)

1. State whether the following statements are true or false. Give reasons for your answers. (6)

   a) The function $f : \mathbf{R}^3 \to \mathbf{R}$, given by $f(x, y, z) = |x| + |y| + |z|$ is differentiable at $(2,3,-1)$.

   b) The function $f(x, y) = \max\left\{\frac{y}{x}, x\right\}$ is a homogeneous function on $\mathbf{R}^2$.

   c) The domain of the function f/g where $f(x, y) = 2xy$ and $g(x, y) = x^2 + y^2$ is $\mathbf{R}^2$.

2. a) Express the following surfaces in spherical coordinates (2)

   i) $xz = 3$
   ii) $x^2 + y^2 - z^2 = 1$

   b) Find the cylindrical coordinates of the points where the Cartesian coordinates are (2)

   i) $(6,6,8)$
   ii) $(\sqrt{2},1,1)$

   c) Show that the closed sphere with centre $(2,3,7)$ and radius 10 in $\mathbf{R}^3$ is contained in the open cube $P = \{(x, y, z) : |x-2| < 11, |y-3| < 11, |z-7| < 11\}$. (3)

   d) Check whether the limit of the function $f(x, y) = \dfrac{3x^3 y}{x^6 + 2y^2}$ exists as $(x, y) \to (0,0)$. (3)

   e) Find the two repeated limits of the function $f(x, y) = \left(\dfrac{y-x}{y+x}\right)\left(\dfrac{1+x^2}{1+y^2}\right)$ at $(0,0)$. Does the simultaneous limit of f exist as $(x, y) \to (0,0)$? Give reasons for your answer. (3)

3. a) Check whether the function

$$f(x, y) = \begin{cases} \dfrac{4x^2 y}{4x^4 + y^2}, & (x, y) \neq (0,0) \\ 0 \quad , & (x, y) = (0,0) \end{cases}$$

   is continuous at $(0,0)$. (3)

b) Find $f_x(0,0)$ and $f_x(x,y)$, where $(x,y) \neq (0,0)$ for the following function

$$f(x,y) = \begin{cases} \dfrac{xy^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Is f_x continuous at $(0,0)$? Justify your answer. (4)

c) Compute f_{xy} and f_{yx} for the function $f(x,y) = e^{x+y}\sin x + 9x^2 + 2xy$ at $(1,2)$. (2)

d) Find $\dfrac{dz}{dt}$ for $z = x^2y + 4y^2$ where $x = \cos t$ and $y = \sin t$ using the chain rule. (2)

Part B (40 Marks)

4) State whether the following statements are true or false. Justify yourself with the help of a short proof or a counter example.

i) $y' + P(x)\,y = Q(x)\,y^n$ is a linear equation for integral values of n.

ii) $y = 0,$ is a singular solution of the differential equation $27y - 8\left(\dfrac{dy}{dx}\right)^3 = 0$

iii) Equation $x^2(y - px) = yp^2$ is reducible to clairaut's form (6)

5) a) Find the value of b for which the equation

$$(ye^{2xy} + x)\,dx + bxe^{2xy}dy = 0$$

is exact, and hence solve it for that value of b (3)

b) Find the solution of the Riccati equation

$$\frac{dy}{dx} = \frac{2\cos^2 x - \sin^2 x + y^2}{2\cos x}; \quad y_1(x) = \sin x$$ (3)

6) a) Solve the differential equation

$$\frac{dy}{dx} + xy = y^2 e^{x^2/2}\sin x$$ (3)

b) Given that $y_1(x) = x^{-1}$ is one solution of the differential equation

$$2x^2y'' + 3xy' - y = 0, \; x > 0,$$

find a second linearly independent solution of the equation (3)

7) a) Solve the differential equation

$$\frac{dy}{dx}+\left(\frac{x}{1-x^2}\right)y = x\sqrt{y},\ y(0)=1$$ (3)

b) A wet porus substance in the open air loses its moisture at a rate proportional to the moisture content, if a sheet hung in the wind loses half its moisture during the first hour, then find the time when it has lost 95% moisture provided the weather conditions remain the same. (3)

8) a) Solve, using the method of variation of parameters

$$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$$ (3)

b) Solve the following initial value problem

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=-6\sin 2x-18\cos 2x$$

$$y(0)=2,\ y'(0)=2$$ (4)

9) a) Identify the type of the differential equation $y = xy' + 1 - \ln y'$ and hence solve it (2)

b) Using the method of undermined coefficients, find the general solution of the differential equation

$$y^{iv}-2y'''+2y''=3e^{-x}+2e^{-x}x+e^{-x}\sin x$$ (3)

c) A simple series circuit has an inductor of 1 henry, a capacitor of 10^{-6} farads and a resistor of 1000 ohms. The initial charge on the capacitor is zero. If a 12 volt battery is connected to the circuit and the circuit is closed at $t=0$, find the charge on the capacitor 1 second later and the steady state charge. (4)

Part C (30 Marks)

10) State whether the following statements are true or false. Justify your answer with the help of a short proof or a counter example.

i) Equation $\cos(x+y)\,p+\sin(x+y)\,q = z^2+z$ is a quasi-linear equation.

ii) The solution of PDE $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=z^2$ is $z=-[y+f(x-y)]$

iii) $\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}-\left(\frac{\partial z}{\partial y}\right)^2=0$ is a non-linear PDE. (6)

11) a) Find the differential equation of the space curve in which the two families of surfaces $u = x^2 - y^2 = c_1$ and $v = y^2 - z^2 = c_2$ intersect. (2)

b) Find the general integral of the equation

$(x - y)\,p + (y - x - z)\,q = z$

and a particular solution through the circle $z = 1, x^2 + y^2 = 1$ (4)

12) Verify that the equations

i) $z = \sqrt{2x + a} + \sqrt{2y + b}$ and

ii) $z^2 + \mu = 2(1 + \lambda^{-1})(x + \lambda y)$

are both complete integrals of the PDE $z = \dfrac{1}{p} + \dfrac{1}{q}$. Also show that the complete integral (ii) is the envelope of one parameter sub-system obtained by taking $b = \dfrac{-a}{\lambda} - \dfrac{\mu}{1 + \lambda}$ in the solution (i) (6)

13) a) Verify that the total differential equation

$yz\,dx + (x^2 y - zx)\,dy + (x^2 z - xy)\,dz = 0$

is integrable and hence find its integral. (3)

b) Show that $2z = (ax + y)^2 + b,$ where a, b are arbitrary constants is a complete integral of $px + qy - q^2 = 0.$ (3)

14) a) Find the partial differential equation arising from $\phi\left[\dfrac{z}{x^3}, \dfrac{y}{z}\right] = 0,$ where ϕ is an arbitrary function of the arguments. Also find the general solution of the PDE obtained. (4)

b) Given that $z = ax + \sqrt{a^2 - 4}\,y + c$ is the complete integral of the PDE, $p^2 - q^2 = 4,$ determine its general integral. (2)