BMTE-144

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

(BSCG / BAG)
NUMERICAL ANALYSIS

Valid from 1st January, 2022 to 31st Dec, 2022

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for B.Sc. that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, Part A, Part B, Part C. The maximum marks of all the parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ………………………………………….

NAME: ………………………………………….

ADDRESS: ………………………………………….

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….

STUDY CENTRE: ………………………….….. **DATE:** ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) Solve Part A, Part B and Part C of this assignment, and **submit the complete assignment answer sheets within the due date.**

6) The assignment answer sheets are to be submitted to your Study Centre within the due date. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is **valid from 1st Jan, 2022 to 31st Dec, 2022**. If you have failed in this assignment or fail to submit it by Dec, 2022, then you need to get the assignment for the year 2023, and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the examination form for this course** until you have submitted this assignment.

We wish you good luck.

ASSIGNMENT

Course Code: BMTE-144
Assignment Code: BMTE-144/TMA/2022
Maximum Marks: 100

Part-A (Based on Blocks 1 and 2 of the course)

1. a) The equation $x^3 - x - 1 = 0$ has a positive root in the interval $]1, 2[$. Write a fixed point iteration method and show that it converges. Starting with initial approximation $x_0 = 1.5$ find the root of the equation correct to three decimal places. (4)

   b) Find an appropriate root of $x^3 + 2x^2 - 5 = 0$ in $[1, 2]$ with 10^{-5} accuracy by

   i) Newton Raphson Method

   ii) Secant Method

   What conclusions can you draw from here about the two methods? (6)

2. a) Using Maclaurin's expansion for $\sin x$, find the approximate value of $\sin\frac{\pi}{4}$ with the error bound 10^{-5}. (3)

   b) Find an approximate value of the positive real root of $xe^x = 1$ using graphical method. Use this value to find the positive real root of $xe^x = 1$ correct to three decimal places by fixed point iteration method. (4)

   c) Using $x_0 = 0$ find an approximation to one of the zeros of $x^3 - 4x + 1 = 0$ by using Birge-Vieta Method. Perform two iterations. (3)

3. a) The iteration method

$$x_{n+1} = \frac{1}{8}\left[6x_n + \frac{3N}{x_n} - \frac{x_n^3}{N}\right], \ n = 0, 1, 2$$

   where N is positive constant, converges to some quantity. Determine this quantity. Also find the rate of convergence of this method. (4)

   b) Solve the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 + x_4 &= 3 \\ x_1 + 2x_2 + x_4 &= 2 \\ 2x_1 + 3x_2 + x_3 - x_4 &= 1 \\ x_1 - 2x_2 - x_3 + 4x_4 &= 5 \end{aligned}$$

   using Gauss elimination method with pivoting. (3)

   c) Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 2 \\ -2 & 3 & -5 \\ 1 & 2 & 4 \end{bmatrix}$ using Gauss Jordan Method. (3)

Part-B (Based on Blocks 2 and 3 of the course)

4) a) Solve the system of equations

$$
\begin{aligned}
8x_1 - x_2 + 2x_3 &= 4 \\
-3x_1 + 11x_2 - x_3 + 3x_4 &= 23 \\
-x_2 + 10x_3 - x_4 &= -13 \\
-2x_1 + x_2 - x_3 + 8x_4 &= 13
\end{aligned}
$$

with $x^{(0)} = [0\ 0\ 0\ 0]^T$, by using the Gauss Jacboi and Gauss Seidel method. The exact solution of the system is $x = [1\ 2\ -1\ 1]^T$. Perform the required number of iterations so that the same accuracy is obtained by both the methods. What conclusions can you draw from the results obtained? (5)

b) Starting with $x^{(0)} = [1\ 1\ 1]^T$, find the dominant eigenvalue and corresponding eigenvector for the matrix $A = \begin{bmatrix} 4 & -1 & 1 \\ 4 & -8 & 1 \\ -2 & 1 & 5 \end{bmatrix}$ using the power method. (5)

5. a) The solution of the system of equations $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ is attempted by the Gauss Jacobi and Gauss Seidel iteration schemes. Set up the two schemes in matrix form. Will the iteration schemes converge? Justify your answer. (3)

b) Solve the following linear system $Ax = b$ of equations with partial pivoting

$$
\begin{aligned}
x_1 - x_2 + 3x_3 &= 3 \\
2x_1 + x_2 + 4x_3 &= 7 \\
3x_1 + 5x_2 - 2x_3 &= 6.
\end{aligned}
$$

Store the multipliers and also write the pivoting vectors. (4)

c) Find the minimum number of intervals required to evaluate $\int_0^1 e^{-x^2} dx$ with an accuracy of $\frac{1}{2} \times 10^{-4}$, by using the Trapezoidal rule. (3)

6. a) From the following table, find the number of students who obtained less than 45 marks. (4)

Marks	No. of Students
30-40	31
40-50	42
50-60	51
60-70	35
70-80	31

b) Calculate the third-degree Taylor polynomial about $x_0 = 0$ for $f(x) = (1+x)^{1/2}$. (3)

c) Use the polynomial in part (a) to approximate $\sqrt{1.1}$ and find a bound for the error involved. (2)

d) Use the polynomial in part (a) to approximate $\int_0^{0.1} (1+x)^{1/2}\, dx$. (1)

Part-C (Based on Blocks 3 and 4 of the course)

7. a) Using $\sin(0.1) = 0.09983$ and $\sin(0.2) = 0.19867$, find an approximate value of $\sin(0.15)$ by using Lagrange interpolation. Obtain a bound on the truncation error. (3)

b) Consider the following data

x	1.0	1.3	1.6	1.9	2.2
f(x)	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

Use Stirling's formula to approximate $f(1.5)$ with $x_0 = 1.6$. (3)

c) Solve the I.V.P., $y' = -y + t + 1,\ 0 \le t \le 1,\ y(0) = 1$ using R-K method of $0(h^4)$ with $h = 0.1$ and obtain the value of $y(0.2)$. Also find the error at $t = 0.2$, if the exact solution is $y(t) = t + e^{-t}$. (4)

8. a) The position $f(x)$ of a particle moving in a line at various times x_k is given in the following table. Estimate the velocity and acceleration of the particle at $x = 1.2$ (3)

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
f(x)	2.72	3.32	4.06	4.96	6.05	7.39	9.02

b) A solid of revolution is formed by rotating about the x-axis the area bounded between $x = 0,\ x = 1$ and the curve given by the table

x	0	0.25	0.5	0.75	1.0
f(x)	1.0	0.9896	0.9587	0.9089	0.8415

Find the volume of the solid so formed using

i) Trapezodial rule ii) Simpson's rule (3)

c) Take 10 figure logarithm to base 10 from $x = 300$ to $x = 310$ by unit increment. Calculate the first derivative of $\log_{10} x$ when $x = 310$. (4)

9. a) For the table of values of $f(x) = xe^x$ given by

x	1.8	1.9	2.0	2.1	2.2
f(x)	10.8894	12.7032	14.7781	17.1489	19.8550

Find $f''(2.0)$ using the central difference formula of $0(h^2)$ for $h = 0.1$ and $h = 0.2$. Calculate T.E. and actual error. (3)

b) Suppose f_n denotes the value of $f(t)$ at $t = t_n$. If $f(t) = t^3$ then find the value of $\dfrac{(f_{n+1} - 2f_n + f_{n-1})}{h^2}$. (2)

c) Use Runge-Kutta method of order four to solve $y' = x + y$. Start with $x = 1,\ y = 0$ and carry to $x = 1.5$ with $h = 0.1$. (3)

d) Find the solution of the difference equation $y_{k+2} - 4y_{k+1} + 4y_k = 0,\ k = 0, 1, \ldots$. Also find the particular solution when $y_0 = 1$ and $y_1 = 6$. (2)

10. a) Obtain an approximate value of $\int_0^1 \frac{dx}{1+x^2}$ using composite Simpson's rule with $h = 0.25$ and $h = 0.125$. Find also the improved value using Romberg integration. (4)

b) Determine the spacing h in a table of equally spaced values for the function $f(x) = (2+x)^4,\ 1 \le x \le 2$, so that the quadratic interpolation in this table satisfies $|\text{error}| \le 10^{-6}$. (3)

c) Determine a unique polynomial $f(x)$ of degree ≤ 3 such that $f(x_0) = 1,\ f'(x_0) = 2,\ f(x_1) = 2,\ f'(x_1) = 3$ where $x_1 - x_0 = h$. (3)