

BMTC-131

ASSIGNMENT BOOKLET

**Bachelor's Degree Programme
(BSCG/BAG)
CALCULUS**

Valid from 1st July, 2021 to 30th June, 2022



**School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068**

(2021-22)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, A, B and C. The maximum marks of all the parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

- 1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:

NAME:

ADDRESS:

.....

.....

COURSE CODE:

COURSE TITLE:

ASSIGNMENT NO.:

STUDY CENTRE: **DATE:**

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

- 2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
- 3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
- 4) Your answers should be precise.
- 5) While solving problems, clearly indicate which part of which question is being solved.
- 6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**
We strongly suggest that you retain a copy of your answer sheets.
- 7) This assignment is valid only upto **June, 2022**. If you have failed in this assignment or fail to submit it by June, 2022, then you need to get the assignment for the year 2022-2023 and submit it as per the instructions given in the programme guide.
- 8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: BMTC-131
Assignment Code: BMTC-131/TMA/2021-22
Maximum Marks: 100

PART – A (35 marks) (Based on Blocks 1 and 2 of the course)

1. Which of the following statements are true, and which are false? Give reasons for your answers in the form of a short proof or a counterexample. (10)
- i) The function $f: \mathbf{R} \rightarrow \mathbf{R}$ defined as $f(x) = (x - 7)(x^3 + 11)$ is an odd function.
 - ii) All the cube roots of i in \mathbf{C} are $z_1 = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$, $z_2 = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$ and $z_3 = \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}$.
 - iii) If $f(x) = 2|x - 1|$ and $g(x) = 3x - 10$, then $f \circ g(1) = 12$.
 - iv) The domain of the function f defined by $f(x) = \sqrt{\frac{3-x}{x-2}}$ is $\mathbf{R} - \{2\}$.
 - v) $(\sqrt{2}, 1, \frac{1}{2}) \in \mathbf{Q} \times \mathbf{Z} \times \mathbf{R}$.
2. a) If a, b and c are the roots of the equation $x^3 - 6x^2 + 10x - 6 = 0$, find the values of $a^2 + b^2 + c^2$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$. (4)
- b) Using the $\varepsilon - \delta$ definition, show that $\lim_{x \rightarrow 2} (3x - 5) = 1$. (4)
- c) Find the square root of $1+i$. (2)
3. a) Find the domain and range of the function f defined by $f(x) = \frac{1}{1 - \sin x}$. (3)
- b) Let $f(x) = \begin{cases} 1 + 2x, & x \leq 0 \\ 3x - 2, & 0 < x \leq 1 \\ 2x^2 - 1, & x > 1 \end{cases}$
- i) Check whether f is discontinuous. If yes, find where?
 - ii) Give a rough sketch of the graph of f . (5)
- c) Express the function g , defined by $g(x) = \frac{1}{\sqrt{x + \sqrt{x}}}$, as a composition of three functions. (2)
4. Find the following limits: (5)
- (i) $\lim_{x \rightarrow 2} \frac{x^2 + 4x - 1}{x^2 - 2x}$
 - (ii) $\lim_{x \rightarrow 0} x^2 \cos \frac{1}{x}$

PART – B (40 Marks)
(Based on Blocks 3 and 4 of the course)

5. a) If $f(x) = \sin^{-1} x$, show that $(1 - x^2)f''(x) - xf'(x) = 0$. Hence, prove that $f^{(n+2)}(0) = n^2 f^{(n)}(0)$. (4)
- b) Find the maximum height of the curve $y = 4 \sin x - 3 \cos x$ above the x-axis. (3)
- c) Verify Rolle's theorem for f on $[-1, 1]$ defined by $f(x) = x^4 - 4x^2 + 7$. (3)
6. a) Find the approximate value of $\sqrt[3]{0.99}$ upto 4 places of decimal. (4)
- b) Differentiate y w.r.t. x in the following cases: (6)
- i) $y = \sqrt{\sin(\sqrt{x})}$
- ii) $y = e^{\sin x} + x \ln x$
7. Which of the following statements are true and which are false? Give reasons for your answers, in the form of a short proof or a counter example.
- i) The function f defined by $f(x) = \tan(2x)$ is a periodic function with period π .
- ii) The function $f: \mathbf{R} \rightarrow \mathbf{R}$, defined by $f(x) = 1 - |x|$ is differentiable at $x=1$.
- iii) The function $f: [3,4] \rightarrow \mathbf{R}$ defined by $f(x) = x^2 - x$ is monotonic in its domain.
- iv) Every continuous function is differentiable.
- v) Every curve over \mathbf{R} has a point of inflection. (10)
8. Trace the curve $y = \frac{8}{4-x^2}$, and state all the properties you use to trace it. (10)

PART – C (25 Marks)
(Based on Block 5 of the course)

9. Integrate the following functions w.r.t. x : (9)
- i) $\int \frac{x-1}{x^3-x^2-2x} dx$
- ii) $\int_2^4 x \ln x dx$
- iii) $\int \frac{(\cot^{-1} x)^4}{1+x^2} dx$
10. (a) If $I_n = \int_0^\infty e^{-x} \sin^n(x) dx$, prove that $(1 + n^2)I_n = n(n - 1)I_{n-2}$ for $n \geq 2$. (5)

b) Find the length of the curve given by $x = t^3, y = 2t^2$ in $0 \leq t \leq 1$. What is the slope of the curve at $t = \frac{1}{2}$. (5)

11. Which of the following statements are true? Give reasons for your answers, in the form of a short proof or a counter example. (6)

i) $\frac{d}{dx} \left[\int_{x^2}^{\frac{\pi}{2}} e^{2 \cos t} dt \right] = -2x e^{2(\cos x^2)}$

ii) $|x|$ is integrable in $[-2, 2]$.

iii) $\int_a^b x dx = \frac{b^2 - a^2}{2}$.