MTE-12

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

LINEAR PROGRAMMING

(Valid from 1st January, 2021 to 31st December, 2021)

It is compulsory to submit the assignment before filling the exam form.

School of Sciences
Indira Gandhi National Open University
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(2021)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….

STUDY CENTRE: ……………………………. **DATE:** ……………………………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2021**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.
We wish you good luck.

ASSIGNMENT

Course Code: MTE-12
Assignment Code: MTE-12/TMA/2021
Maximum Marks: 100

1. Which of the following statements are true? Give a short proof or a counter example in support of your answer. (10)

   (i) For any two square matrices A and B, $AB = BA$.

   (ii) If the following table is obtained in the intermediate stage while solving an LPP by the Simplex method, then the LPP has an unbounded solution:

		-1	-2	0	0	0
1	x_1	1	2	-1	0	1
0	x_4	0	3	-1	1	2
		0	4	-1	0	1

   (iii) The number of basic variables in a feasible solution of a transportation problem with m sources and n destinations is mn.

   (iv) An optimal assignment of the assignment problem with cost matrix C is also an optimal assignment of the assignment problem with cost matrix C^t.

   (v) $(1,2)$ is an optimal solution to the following LPP:

$$\text{Max } Z = 2x_1 + 4x_2 \text{ subject to}$$
$$x_1 + 2x_2 \leq 5$$
$$x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

2. Solve the (4×3) game with pay off matrix. (10)

$$A = \begin{bmatrix} 8 & 5 & 8 \\ 8 & 6 & 5 \\ 7 & 4 & 5 \\ 6 & 5 & 6 \end{bmatrix}$$

   At each stage, clearly explain the steps involved.

3. Solve the following LPP by the two-phase simplex method. (10)

   Max $Z = x_1 + x_2 - x_3$

   Subject to $4x_1 + x_2 + x_3 = 4$

$$3x_1 + 2x_2 - x_4 = 6$$
$$x_1, x_2, x_3 \geq 0$$

4. (a) For the transportation problem given below, check whether the given basic feasible solution is optimal. If not, modify the given solution and find an optimal solution and the optimal value for the problem. (5)

6	1 (25)	9	3 (45)	70
11	5 (5)	2 (50)	8	55
10 (85)	12 (5)	4	7	90
85	35	50	45	

(b) For the following matrix game, write down the equivalent LPPs for solving the game.

$$\begin{array}{cc} & B \\ A = & \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix} \end{array}$$
(5)

5. A businessman needs five cabinets, 12 desks and 18 shelves cleaned out. He has two part time employees, Rashid and Ruby. Ruby can clean one cabinet, three desks and three shelves in a day while Rashid can clean one cabinet, two desks and 6 shelves in one day. Rashid is paid Rs. 22 per day and Ruby is paid Rs. 25 per day. In the order to minimize the cleaning costs, for how many days should Rashid and Ruby be employed? Formulate the problem as a linear programming problem and find its solution by the graphical method. (10)

6. (a) A company has 5 jobs to be processed by 5 mechanics. The following table gives the return in rupees when the i^{th} job is assigned to the j^{th} mechanic. $(i, j = 1, 2, \ldots, 5)$. How should the jobs be assigned to the mechanics so as to maximize the overall return? (5)

		Jobs				
		1	**2**	**3**	**4**	**5**
	1	22	28	30	18	30
Mechanics	2	30	34	18	11	26
	3	31	17	23	20	27
	4	12	28	31	26	26
	5	19	23	30	25	29

(b) Write the dual of the following LPP after reducing it to canonical form. (4)

Min $Z = 3x_1 + 4x_2 + 3x_3$

Subject to

$2x_1 + 4x_2 = 12$

$5x_1 + 3x_3 \geq 11$

$6x_1 + x_2 \geq 8$

$x_1, x_2, x_3 \geq 0$

7. (a) If $A = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $C = [1 \quad 2]$, (2)

compute $A^t B$, AC $B^t A + CB$, whenever defined. If you think any of these are not defined, give reasons for saying so.

(b) Is the set of vectors $\{(1,2,3), (3,4,1), (2,3,2)\}$ linearly independent? Give reasons for the answer. (3)

6	4	1	5	14
8	9	2	7	16
4	3	6	2	5
6	10	15	4	

(c) Find an initial basic feasible solution to the following transportation problem by the North-West corner method. Verify whether your solution is optimal. (5)

8. (a) A manufacturer has two products P_1 and P_2, both of which are produced in two steps by machines M_1 and M_2. The process time per hundred for the products on the machines (6)

	M_1	M_2	Profit (in thousand Rs. per 100 units)
P_1	4	5	10
P_2	5	2	5
Available hours	100	80	

The manufacturer can sell as much as he can produce of both products. Formulate the problem as LP model. Determine optimum solution, using simplex method.

(b) Using graphical method, solve the game whose pay-off matrix is given as: (4)

Player B

		I	II	III	IV
Player A	I	1	3	-3	7
	II	2	5	4	-6

9. (a) Show that the set $S = \{(x, y) \mid 3x^2 + 5y^2 \leq 15\}$ is convex. (5)

(b) Consider the system of equations (5)

$$2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

A feasible solution is $x_1 = 2, x_2 = 3, x_3 = 1$. Reduce this feasible solution to a basic feasible solution.

10. Find the initial basic feasible solution of the following transportation problem using North-West corner method: (10)

	P_1	P_2	P_3	P_4	Requirement
M_1	19	11	23	11	11

M_2	15	16	12	21	13
M_3	30	25	16	39	19
Availability	6	10	12	15	113

Also, find the optimal solution.