MTE-09

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

REAL ANALYSIS

Valid from 1st January 2021 to 31st December 2021

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2021)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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……………………………………………

COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: …………………………….

STUDY CENTRE: ……………………….….. DATE: ……………………….………………...

PLEASE FOLLOW THE FORMAT ABOVE STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of the very thin variety) for writing your answers.
3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2021. If you have failed in this assignment or fail to submit it by December, 2021, then you need to get the assignment for the year 2022 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

ASSIGNMENT

Course Code: MTE-09
Assignment Code: MTE-09/TMA/2021
Maximum Marks: 100

1. Are the following statements true or false? Give reasons for tour answers. (10)

   a) -2 is a limit point of the interval $]-3,2]$.

   b) The series $\frac{1}{2}-\frac{1}{6}+\frac{1}{10}-\frac{1}{4}+\cdots$ is divergent.

   c) The function, $f(x)=\sin^2 x$ is uniformly continuous in the interval $[0,\pi]$.

   d) Every continuous function is differentiable.

   e) The function f defined on $\mathbb{R}$ by

   $$f(x)=\begin{cases} 0, & x \text{ is rational} \\ 2, & x \text{ is irrational} \end{cases}$$

   Is integrable in the interval $[2,3]$.

2) a) Prove that the union of two closed sets is a closed set. Give an example to show that union of an infinite number of closed sets need not be a closed set. (4)

   b) Examine the function $f:\mathbb{R}\to\mathbb{R}$ defined by

   $$f(x)=\begin{cases} \frac{1}{6}(x+1)^3 & x\neq 0 \\ \frac{5}{6} & x=0 \end{cases}$$

   for continuity on $\mathbb{R}$. If it is not continuous at any point of $\mathbb{R}$, find the nature of discontinuity there. (4)

   c) Find $\lim\limits_{x\to 0}\frac{1-\cos^2}{x^2\sin x^2}$. (2)

3) a) Using the principle of mathematical induction, prove that $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number, $\forall n\in N$. (4)

   b) Show that there is no real number, k for which the equation, $x^4-3x^2+k=0$ has two distinct roots in the interval $[2,3]$. (3)

c) Let $f:[-3,3]\to\mathbb{R}$ be defined by $f(x)=5(x)+x^3$, where $[x]$ denotes the greatest integer $\leq x$. Show that this function is integrable. (3)

4. a) Prove that the function f defined by

$$f(x)=\begin{cases}2, & \text{if } x \text{ is irrational}\\ -2, & \text{if } x \text{ is rational}\end{cases}$$

is discontinuous, $\forall\, x\in\mathbb{R}$, using the sequential definition of continuity. (4)

b) Examine the convergence of the following series: (4)

i) $\dfrac{3\times4}{5^2}+\dfrac{5\times6}{7^2}+\dfrac{7\times8}{9^2}+\cdots$

ii) $1+4x+4^2x^2+4^3x^3+\cdots(x>0)$

c) Prove that the set of integers is countable. (2)

5. a) Prove that (4)

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{2n-1}}+\frac{1}{\sqrt{4n-2^2}}+\frac{1}{\sqrt{6n-3^2}}+\cdots+\frac{1}{n}\right]=\frac{\pi}{2}$$

b) Prove that the sequence $\left\{\dfrac{a_n}{n}\right\}$ is convergent where $\{a_n\}$ is a bounded sequence. (3)

c) Prove that continuous function of a continuous function is continuous. (3)

6. a) Examine the function, $f(x)=(x+1)^3(x-3)^2$ for extreme values. (4)

b) Show that the series $\displaystyle\sum_{n=1}^{\infty}\frac{x}{1+n^2x^2}$ is uniformly convergent in $[\alpha,1]$ for any $\alpha>0$. (4)

c) Give an example of an infinite set with finite number of limit points, with proper justification. (2)

7. a) Show that (4)

i) $\displaystyle\lim_{x\to\infty}\left(\frac{x-3}{x-1}\right)^x=\frac{1}{e^2}$

ii) $\lim_{x \to \frac{5}{3}} \frac{1}{(3x+5)^2} = \infty$

b) For the function, $f(x) = x^2 - 2$ defined over $[1,5]$, verify : $L(P, f) \leq U(-P, f)$ where P is the partition which divides $[1,5]$ into four equal intervals. (3)

c) Let $\{a_n\}$ be a sequence defined as $a_1 = 3, a_{n+1} = \frac{1}{5} a_n$ show that $\{a_n\}$ converges to zero. (3)

8. a) Using the sequential definition of the continuity, prove that the function f, defined by: (4)

$$f(x) = \begin{cases} 3, & \text{if } x \text{ is irrational} \\ -3, & \text{if } x \text{ is rational} \end{cases}$$

is discontinuous at each real number.

b) Show that on the curve, $y = 3x^2 - 7x + 6$, the chord joining the points whose abscissa are $x = 1$ and $x = 2$, is parallel to the tangent at the whose abscissa is $x = \frac{3}{2}$. (4)

c) Give an example of a series $\sum a_n$ such that $\sum a_n$ is not convergent but the sequence (a_n) converges to 0. (2)

9. a) Test the series: (3)

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sin nx}{n\sqrt{n}}$$

for absolute and conditional convergence.

b) Check whether the function f given by: (4)

$$f(x) = (x-4)^3 (x+1)^2$$

has local maxima and local minima.

c) Check, whether the collection G, given by: (3)

$$G' = \left\{ \left] \frac{1}{n+2}, \frac{1}{n} \right[: n \in N \right\}$$

is an open cover of $]0,1[$.

10. a) State Bonnet's mean value theorem for integrals. Apply it to show that: (4)

$$\left|\int_3^5 \frac{\cos x}{x}\,dx\right| \le \frac{2}{3}$$

b) Show that the sequence (a_n), where $a_n \frac{n}{n^2+4}$ is monotonic. Is (a_n) a Cauchy sequence? Justify your answer. (4)

c) Check whether the intervals $[2,5]$ and $[7,10]$ are equivalent or not. (2)