MTE-06

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

ABSTRACT ALGEBRA

(Valid from 1st January, 2021 to 31st December, 2021)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2021)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….……………….. .

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2021**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying Blocks 1, 2 and 3)

Course Code: MTE-06
Assignment Code: MTE-06/TMA/2021
Maximum Marks: 100

1) Which of the following statements are true? Justify your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so.)

i) If A and B are two sets such that $A \subseteq B$, then $A \times B = B$.

ii) If S is the set of people on the rolls of IGNOU in 2016 and T is the set of real numbers lying between 2.5 and 2.55, then $S \cup T$ is an infinite set.

iii) The set $\{x \in \mathbb{Z} | x \equiv 1 (\text{mod}\, 30)\}$ is a group with respect to multiplication (mod 30).

iv) If G is a group with an abelian quotient group G/N, then N is abelian.

v) There is a group homomorphism f with Ker $f \simeq \mathbb{R}$ and $\text{Im}\, f \simeq \{0\}$.

vi) There is a 1 – 1 correspondence between the odd permutations of S_{35} and the even permutations of S_{35}.

vii) If R is a ring such that $a = -a \; \forall \, a \in R$, then R is Boolean.

viii) Given any ring R, there is an ideal I of R such that R/I is commutative.

ix) If S is an ideal of a ring R and f a ring homomorphism from R to a ring R', then $f^{-1}(f(S)) = S$.

x) ‘ring’, as we now define it, was first presented to us by Dedekind. (20)

2) a) Prove that $2^n > 4n$ for $n \geq 5$. (3)

b) Give an example, with justification, of a function with domain $\mathbb{Z} \setminus \{2,3\}$ and co-domain $\mathbb{N}$. Is this function 1 – 1? Is it onto? Give reasons for your answers. (4)

c) Give a set of cardinality 5 which is a subset of $\mathbb{Z} \setminus \mathbb{N}$. (1)

d) Check whether the relation $R = \{(x, y) \in \mathbb{N} \times \mathbb{N} | xy \text{ is the square of an integer}\}$ is an equivalence relation on $\mathbb{N}$. (2)

3) a) The table below is a Cayley table for the group $(\{e,a,b,c,d\},*)$. Fill in the blanks.

*	e	a	b	c	d
e	e	-	-	-	-
a	-	b	-	-	e
b	-	c	d	e	-
c	-	d	-	a	b
d	-	-	-	-	-

(3)

b) Let G be a finite group. Show that the number of elements g of G such that $g^3 = e$ is odd, where e is the identity of G. (3)

c) Check if $\left\{\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \middle| \, a,b,c \in \mathbb{R}\right\}$ is an abelian group with respect to matrix multiplication. (4)

4) a) Check whether $H = \{x \in \mathbb{R}^* \mid x = 1 \text{ or } x \text{ is irrational}\}$ and $K = \{x \in \mathbb{R}^* \mid x \geq 1\}$ are subgroups of $(\mathbb{R}^*, .)$. (3)

b) Let $U(n) = \{m \in \mathbb{N} \mid (m,n) = 1, m \leq n\}$. Then U(n) is a group with respect to multiplication modulo n. Find the orders of <m> for each $m \in U(10)$. (3)

c) Find $Z(D_{2n})$, where D_{2n} is the dihedral group with 2n elements,

i) when n is an odd integer;

ii) when n is an even integer. (4)

5. a) Obtain the left cosets of $V_4 = \{e, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$ in A_4. (3)

b) If G is a group with o(g) < 100 and G has subgroups of order 10 and 25, what is the order of G? (2)

c) Show that in a group G of odd order, the equation $x^2 = e$ has a unique solution. Further, show that $x^2 = g$ has a unique solution $\forall g \in G, g \neq e$. (5)

6. a) Check whether the subgroup of reflections and subgroup of rotations in D_{2n} is normal in D_{2n} or not.
(Note that D_{2n} is the group of symmetries of an n-gon.) (3)

b) Prove, by contradiction, that A_4 has no subgroup of order 6. (3)

c) What is the order of

i) 14 in $\mathbb{Z}_{24}/\langle \overline{8} \rangle$?

ii) $(\mathbb{Z}_{10} \oplus U(10))/\langle (2,9) \rangle$? (4)

7. a) Can there be a homomorphism from $\mathbb{Z}_8 \oplus \mathbb{Z}_2$ onto $\mathbb{Z}_4 \oplus \mathbb{Z}_4$? Give reasons for your answer. (2)

b) Define $f:\mathbb{Z} \to \mathbb{Z}_m \times \mathbb{Z}_n : f(x) = (x \bmod m, x \bmod n), m, n \in \mathbb{N}$.

i) If (m, n) = (3, 4), find Ker f.
ii) If (m, n) = (6, 4), find Ker f.
iii) What can you generalize about Ker f from (i) and (ii)? (3)

c) Use FTH to determine all homomorphic images of D_8, upto isomorphism. (5)

8. a) If U(R) denotes the group of units of a ring R, show that $U(R_1 \times R_2) = U(R_1) \times U(R_2)$ for rings R_1 and R_2. (2)

b) Let R be a ring, I an ideal of R, J an ideal of I. Show that if J has a unity, then J is an ideal of R. Also give an example to show that if J does not satisfy this condition it need not be an ideal of R. (5)

c) Let F be the ring of all functions from $\mathbb{R}$ to $\mathbb{R}$ w.r.t. pointwise, addition and multiplication. Let S be the set of all differentiable functions in F. Check whether S is

i) a subring of F,
ii) an ideal of F. (3)

9. a) Prove that every ideal I of a ring R is the kernel of a ring homomorphism of R. (2)

b) Prove that $\mathbb{Z}[\sqrt{2}]$ is isomorphic to $H = \left\{ \begin{bmatrix} a & 2b \\ b & a \end{bmatrix} \middle| a, b \in \mathbb{Z} \right\}$ as rings. (3)

c) Let R and S be rings and f : R → S be a homomorphism. If x is an idempotent in R, show that f(x) is an idempotent in S. Hence, or otherwise, determine all ring homorphisms from $\mathbb{Z} \times \mathbb{Z}$ to $\mathbb{Z}$. (5)