MTE-05

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

Bachelor's Degree Programme
Analytical Geometry
(1st January, 2021 to 31st December, 2021)

- **It is compulsory to submit the Assignment before filling in the Term-End Examination form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives(56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from elective courses from Life Sciences, Chemistry and Physics discipline must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, at least 6 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2021)

Dear Student,
Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 30%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1. On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. : .....
NAME : .....
ADDRESS : .....
.....
.....
COURSE CODE : ....
COURSE TITLE : .....
STUDY CENTRE : ..... **DATE** .....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2. Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3. Leave a 4 cm margin on the left, top and bottom of your answer sheet.
4. Your answers should be precise.
5. While solving problems, clearly indicate which part of which question is being solved.
6. This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**
7. This assignment is valid only up to 31st December, 2021. If you fail in this assignment or fail to submit it by 31st December, 2021, then you need to get the assignment for the year 2022 and submit it as per the instructions given in the Programme Guide.
8. **You cannot fill the Exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**
9. **We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

Assignment

Course Code: MTE-05
Assignment Code: MTE-05/TMA/2021
Maximum Marks: 100

1. Which of the following statements are true and which are false? Give reasons for your answer. (20)

   i) The equation $r = a\cos\theta + b\sin\theta$ represents a circle.

   ii) If $1, 1/2, 0$ are direction ratios of a line, then the line makes an angle of 90° with the x-axis, an angle of 60° with the y-axis, and is parallel to the z-axis.

   iii) The intersection of a plane and a cone can be a pair of lines.

   iv) If a cone has three mutually perpendicular generators then its reciprocal cone has three mutually perpendicular tangent planes.

   v) The equations $2x^2 + y^2 + 3z^2 + 4x + 4y + 18z + 34 = 0, 2x^2 - y^2 = 4y - 4y - 4x$ represent a real conic.

   vi) $4x^2 - 9y^2 + z^2 + 36 = 0$ represents a hyperboloid of one sheet.

   vii) The intersection of any plane with an ellipsoid is an ellipse.

   viii) No plane passes through the points $(1,2,3), (1,-1,0)$ and $(1,1,2)$.

   ix) The circle with centre $(a,0)$ and radius a, where $a > 0$, touches all the sides of the square $x = 0, x = a, y = \pm a$.

   x) If the projection of a line segment AB on a line L is 0, then AB lies in L.

2. a) Find the equation of the normal to the parabola $y^2 + 4x = 0$ at the point where the line $y = x + c$ touches it. (2)

   b) Identify and trace the conic $x^2 - 2xy + y^2 - 3x + 2y + 3 = 0$. (4)

   c) Prove that the plane $2x - 3y + 6z = 6$ touches the conicoid $4x^2 - 9y^2 + 36z^2 = 36$. Find the point of contact. (3)

   d) Examine which of the following conicoids are central and which are non-central. Also determine which of the central conicoids have centre at the origin. (6)

      i) $x^2 + y^2 + z^2 + x + y + z = 1$

      ii) $2x^2 + 4xy + xz - x - 3y + 5z + 3 = 0$

      iii) $x^2 - y^2 - z^2 + xy + 4yz + x = 0$

3. a) Obtain the equation of the conic, a focus of which lies at $(2,1)$, the directrix of which is $x + y = 0$ and which passes through $(1,4)$. Also identify the conic. (2)

   b) Find the equations of the spheres which pass through the circle $x^2 + y^2 + z^2 = 9, 2x + 2y - 7 = 0$ and the touch the plane $x - y + z + 3 = 0$ (3)

   c) Find the transformation of the equation $12x^2 - 2y^2 + z^2 = 2xy$ if the origin is kept fixed and the axes are rotated in such a way that the direction ratios of the new axes are $1, -3, 0; 3, 1, 0; 0, 0, 1$. (4)

   d) Identify and trace the conicoid $y^2 + z^2 = x$. Describe its sections by the planes $x = 0, y = 0$ and $z = 0$. (6)

4. a) Find the distance of the point of intersection of the line

$$\frac{x-2}{1}=\frac{y+3}{-1}=\frac{z}{3}$$

and the plane $2x-3y+4z+4=0$ from the origin. (3)

b) Find the equation of the tangent plane to the conicoid $x^2+y^2=kz$ at the point $(k,k,2k)$, where k is a constant. Represent the plane geometrically. Now take different values of k, including both positive and negative, and see how the shape of the conicoid changes. (6)

c) Find the equation of the plane which passes through the line of intersection of the planes $x+y-2z=1$ and $2x+y-4z=3$ and which is perpendicular to the plane $x+y+z=1$. (3)

d) Find the equation of the cylinder with base

$$x^2+y^2+z^2+3x+3y-z=1, x+y+2z=2.$$ (3)

5. a) Find the equation of the normal of the paraboloid $3x^2+4z^2+4y=0$ at the point $(2,-4,1)$. Also find the point where this line again intersects the paraboloid. (3)

b) Let R be the point which divides the line segment joining $P(2,1,0)$ and $Q(-1,3,4)$ into the ratio $1:2$ such that $PR<PQ$. Find the equation of the line passing through R and parallel to the line

$$\frac{x}{2}=y=\frac{z}{2}.$$ (3)

c) Find the equation of the line which passes through the point $(1,\sqrt{3})$ and makes an angle of 30° with the line $x-\sqrt{3}y+\sqrt{3}=0$. (3)

d) Find the vertices, eccentricity, foci and asymptotes of the hyperbola $\frac{x^2}{8}-\frac{y^2}{4}=1$. Also trace it. Under what conditions on λ the line $x+\lambda y=2$ will be tangent to this hyperbola? Explain geometrically. (6)

6. a) Show that the angle between the two lines in which the plane $x-y+2z=0$ intersects the cone $x^2+y^2-4z^2+6yz=0$ is $\tan^{-1}\frac{\sqrt{6}}{7}$. (4)

b) Show that if $ux+vy+wz=p$ is a tangent plane to the paraboloid $ax^2+by^2=2z$, then

$$\frac{u^2}{a}+\frac{v^2}{b}+2pw=0.$$ (3)

c) Prove that the planes
$7x+4y-4z+30=0, 36x-51y+12z+17=0, 14x+8y-8z-12=0$ and $12x-17y+4z-3=0$ form the four faces of a cuboid. (3)

7. a) Find the equation of the right circular cone whose vertex is $(1,-1,2)$, the axis is

$$\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-2}{-2}$$

and the semi-vertical angle is 45°. (2)

b) Prove that the path traced by the foot of the perpendicular from the focus of a parabola on any tangent to the parabola is the tangent at its vertex. (3)

c) Let $S \equiv 4x^2 - 9y^2 - 36 = 0$ and $S' \equiv y^2 - 4x = 0$ be two conics. Under what conditions on k, will the conic $S + kS' = 0$ represent:

i) an ellipse?

ii) a hyperbola? (2)

d) Find the section of the conicoid $\frac{x^2}{2} - \frac{y^2}{3} = 2z$ by the plane $x - 2y + z = 1$. What conic does this section represent? Justify your answer. (3)