MTE-04

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

ELEMENTARY ALGEBRA

(Valid from 1st January, 2021 to 31st December, 2021)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
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New Delhi-110068
(2021)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: …………………………..….. **DATE:** ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2021**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying Blocks 1 and 2)

Course Code: MTE-04
Assignment Code: MTE-04/TMA/2021
Maximum Marks: 100

1) Which of the following statements are true? Justify your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that '{1, padma, blue} is a set' is true, you need to say that this is true because it is a well-defined collection of 3 objects.)

i) The contrapositive of '$\text{not } A \Rightarrow \text{not } B$' is '$A \Rightarrow B$', where A and B are two statements.

ii) Any set can be represented by the listing method.

iii) For any three sets A, B, C in a universal set U, $(A \setminus B) \times C = A \setminus (B \times C)$.

iv) The operation of conjugation is closed on $\mathbb{C}$.

v) If $f(x)$ and $g(x)$ are polynomials for which $f(0) = g(0)$ and $f(1) = g(1)$, then $f(x) = g(x)$.

vi) The solution set of $2x = 1, y + 5 = x, x = y + 3$ is a singleton.

vii) $(1 \;\; 2 \;\; 3 \;\; \ldots\ldots \;\; n)$ is a matrix.

viii) Bunyakovskii and Weirstrass were contemporaries.

ix) The Substitution Method for solving a linear system should be employed when the Elimination Method fails.

x) If a monic polynomial of degree n has n roots in $\mathbb{Z}$, then all its coefficients are in $\mathbb{Z}$. (20)

2) a) Prove that DO, DB and DE are the AM, GM and HM of a and b, as shown in Fig. 1, Unit 6. (6)

b) Give an example, with justification, to show why $0 < a_i < 1$ in Theorem 6, Unit 6. (2)

c) Prove that $\dfrac{[1.2 + 2.3 + \ldots + n(n+1)]}{n(n+3)} \geq \dfrac{n+1}{4}$, for $n \geq 1$. (2)

3) a) Give an example related to the life of a schoolchild, of each of the following. Justify your choice of example also.

i) Two non-empty sets, whose intersection is the null set.

ii) A set with exactly 4 subsets. (5)

b) Give three sets A, B, C such that $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap B \cap C = \emptyset$. Show them in a Venn Diagram also, clearly stating what U is. (3)

c) Prove that if A and B are sets such that $A \times B \neq \emptyset$, then $A \cup C \neq \emptyset$ for any set C. (2)

4) a) Show the geometric representation of the set $\{z \in \mathbb{C} \mid |z+1| = 5\}$. (2)

b) Prove, by contradiction, that if $w, z \in \mathbb{C}$ such that $|w| \leq 1$ and
$w^n z + w^{n-1} z^2 + \ldots + w z^n = 1$, then $|z| > \frac{1}{2}$. (5)

c) If $z = a + 2i$ is a root of $x^2 + 6x + k = 0$, where $a, k \in \mathbb{R}$, find a, as well as the modulus and principal argument of z. Which quadrant does z lie in? (3)

5. a) Obtain the resolvent cubic of $4x^4 + 16x^3 - 17x^2 - 102x - 45 = 0$ using

i) Ferrari's method;
ii) Descartes' method.

Are the cubics you get from (i) and (ii) above the same?

Further, use either method to obtain the roots of the equation.

[**Hint**: To obtain a solution of the resolvent cubic, you can apply Theorem 5 of Unit 6.] (15)

b) Solve the equation $x^5 - 5x^4 - 5x^3 + 25x^2 + 4x - 20 = 0$, given that its roots are of the form $a, -a, b, -b, c$. (5)

6. Solve the following linear systems by the method given alongside each.

Verify your solution also, in each case.

i) $2x - 3y + z = 1$, $x + y + z = 2$, $3x - 4z - 17 = 0$
(by Elimination Method). (5)

ii) $2x - 3y = 1$, $5 - 2y = z$
(by Substitution Method) (3)

iii) $2\alpha + \gamma = 3\beta, \alpha + \beta = 5 + 3\gamma, \alpha - \beta = \pi$
(by Cramer's Rule) (5)

iv) $3x - 5 = y,\ y = 4$ (geometrically) (2)

7. a) A lady bought a plot of land for ₹ 30 lakhs. She wanted to landscape it. So she bought 15 bushes and 18 trees from a nursery for ₹ 975/-. A month later she bought 7 bushes and 5 trees from the same nursery for ₹ 470/-. She paid a gardener ₹ 5000/- to plant them. How much did each bush and tree cost her? (4)

b) A collection of 58 coins, consisting of 25p, 50p, ₹ 1 and ₹ 2 coins are in a bag. The ₹ 1 coins number 5 times that of the 25p coins. The ₹ 2 coins are double the number of ₹ 1 coins, and thrice the number of the 50p coins. If the total value of the coins is ₹ 80/75, how many coins of each kind are there? (4)

c) Create a meaningful problem related to your life that can be represented by the equations x – 4 = y, 2x + y = 5. (2)

8) a) If $A = \begin{vmatrix} 25 & 2.5 & 10 & 1/50 \\ 335 & 573 & \pi & e \\ 25 & 2.5 & 10 & 1/50 \\ e & \pi & 357 & 573 \end{vmatrix}$, find $|A|$. (1)

b) Calculate $\begin{vmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 1 & -1 & 7 \end{vmatrix}, \begin{vmatrix} 0 & 1 & 0 \\ 0 & 4 & 5 \\ 7 & 1 & -1 \end{vmatrix}$ and $\begin{vmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 20 & -20 & 140 \end{vmatrix}$. (2)

c) Write a 3 × 3 square matrix A whose determinant is 75. (2)