**MTE-12**

# ASSIGNMENT BOOKLET

**Bachelor's Degree Programme**
**(B.Sc./B.A./B.Com.)**

# LINEAR PROGRAMMING

**(Valid from 1st January, 2022 to 31st December, 2022)**

**It is compulsory to submit the assignment before filling in the exam form.**

**School of Sciences**
**Indira Gandhi National Open University**
**Maidan Garhi**
**New Delhi-110068**
**(2022)**

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

**Instructions for Formatting Your Assignments**

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

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**ROLL NO.:** …………………………………………

**NAME:** …………………………………………

**ADDRESS:** …………………………………………

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**COURSE CODE:** …………………………….

**COURSE TITLE:** …………………………….

**ASSIGNMENT NO.:** ………………………….…

**STUDY CENTRE:** ………………………..….. **DATE:** ……………………….……………...

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**PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2022**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

**We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

# ASSIGNMENT

## (To be done after studying all the blocks)

**Course Code: MTE-12**
**Assignment Code: MTE-12/TMA/2022**
**Maximum Marks: 100**

1. State if the following statements are true or false. Justify your answer. (10)

   a) The intersection of a finite number of convex sets is not convex.

   b) There exists an extreme point of the common set of feasible solution to a L.P.P. which is not a basic feasible solution.

   c) If value of the $2\times 2$ matrix game $\begin{bmatrix} 1 & 2 \\ p & 4 \end{bmatrix}$ is 4, then $p \geq 4$.

   d) If 10 is added to each of the diagonal entries of the cost matrix in an assignment problem, then the total cost of an optimal assignment for the changed cost matrix will increase by 10.

   e) For the matrix game

$$\begin{array}{cc|ccc} & & & B & \\ & & B_1 & B_2 & B_2 \\ \hline & A_1 & 2 & 1 & -2 \\ A & A_2 & 4 & 4 & 6 \\ & A_3 & -1 & 3 & 5 \end{array}$$

   $(A_2, B_1)$ is a saddle point, but $(A_2, B_2)$ is **not** a saddle point.

2. a) Test for convexity the following sets:

   $S = \{(x, y) \mid x^2 + y^2 \geq 1, y \geq x, y \geq -x\}$

   $S = \{(x, y) \mid x^2 + y^2 \leq 16, x \leq 2, y \geq 2\}$ (6)

   b) Determine all the basic feasible solutions to the equations

   $x_1 + x_2 + 2x_3 = 4$

   $2x_1 - x_2 + x_3 = 2$

   Identify the degenerate basic feasible solutions. (4)

3. a) A firm manufactures pills in two sizes A and B. A contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine. B contains 1 grain of aspirin, 8 grains of bicarbonate and 6 grains of codeine. It requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grains of codeine for providing immediate effect. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate the problem as a standard LPP and solve it graphically. (6)

   b) Write the dual of the following LPP:

   Maximize $z = 2x_1 + x_2$

Subject to $x_1 + 5x_2 \le 10$

$x_1 + 3x_2 \ge 6$

$2x_1 + 2x_2 \le 8$

$x_2 \ge 0$, $x_1$ unrestricted in sign. (4)

4. a) Let $A = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ and c$\begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 5 & 1 \end{bmatrix}$. Compute AB, BC, AC wherever defined. If you think some of these are not defined, give reasons for your answers. (3)

b) Check whether the following LPP has a feasible solution using two-phase method:

Min. $x_1 - 2x_2 - 3x_3$

Such that $-2x_1 + 3x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

$x_1, x_2, x_3 \ge 0.$ (7)

5. a) Determine an initial basic feasible solution to the following transportation problem using matrix minima method and hence find an optimal solution to the problem:

$$\begin{array}{c|cccc|l} & D_1 & D_2 & D_3 & D_4 & \\ \hline O_1 & 1 & 2 & 3 & 4 & 6 \\ O_2 & 12 & 10 & 5 & 7 & 8 \ (5) \\ O_3 & 9 & 11 & 8 & 13 & 10 \\ \hline & 4 & 6 & 8 & 6 & \end{array}$$

(6)

b) Find all values of $k$ for which the vectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} k \\ -k \\ 2 \end{pmatrix}$ are linearly independent. (4)

6. a) A departmental head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task is given in the matrix below:

| Tasks | Men | | | |
|---|---|---|---|---|
| | E | F | G | H |
| A | 18 | 26 | 17 | 11 |
| B | 13 | 28 | 14 | 26 |
| C | 38 | 19 | 18 | 15 |
| D | 19 | 26 | 24 | 10 |

How should the task be allocated, on to a man, so as to minimize the total man-hours? (5)

b) Using dominance solve the game whose pay off matrix is

$$\begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix}$$ (5)

7. a) Solve the following $2 \times 3$ game graphically:

$$\begin{array}{cc} & \text{Player } P_2 \\ \text{Player } P_1 & \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix} \end{array}$$ (6)

b) Consider the following transportation problem:

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 1 | 4 | 30 |
| 3 | 3 | 2 | 1 | 50 |
| 4 | 2 | 5 | 9 | 20 |
| 20 | 40 | 10 | 20 | |

i) Is this transportation problem balanced? Give reasons for your answer.

ii) Obtain a basic feasible solution to the above transportation problem by North-West Corner method. (4)

8. a) Solve by simplex method for following linear programming problem:

Max $z = 2x + y + 2z$

s.t.

$$3 - y + 2z \leq 12$$

$$-2x + 4y \leq 9$$

$$-x + 3y + 8z \leq 15$$

$$x,\ y,\ z \geq 0.$$ (5)

b) Show that the set of vectors $a_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, a_3 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ from a basis for $E^3$. (5)

9. a) Using the initial basic feasible solution for the transportation problem given below, find an optimal solution for the problem. (6)

|  | 1 | 2 | 3 | 4 | Availability |
|---|---|---|---|---|---|
| I | 1 ㊵ | 2 | 3 | 3 ㉚ | 70 |
| II | 2 | 4 ㉘ | 1 | 1 ⑩ | 38 |
| III | 1 | 2 | 3 ㉚ | 2 ② | 32 |
| Requirements | 40 | 28 | 30 | 42 |  |

b) Test the following set for convexity.
$S = \{(x, y) : x + y \leq 8 \text{ or } 2x + y \leq 10, x \geq 0, y \geq 0\}$. (4)

10. a) The following table is obtained in the intermediate stage while solving an LPP by the simplex method.

|  |  | -1 | -2 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|
| 1 | $x_1$ | 1 | 2 | -1 | 0 | 1 |
| 0 | $x_4$ | 0 | 3 | -1 | 1 | 2 |
|  |  | 0 | 4 | -1 | 0 | 1 |

Discuss whether an optimal solution will be exist or not. (4)

b) i) Formulate the dual of the following problem: (6)

Minimise $z = 9x_1 + 12x_2 + 15x_3$

s.t. $2x_1 + 2x_2 + x_3 \geq 10$

$2x_1 + 3x_2 + x_3 \geq 12$

$x_1 + x_2 + 5x_3 \geq 14$

$x_1, x_2, x_3 \geq 0$

ii) Check whether $(2, 2, 2)$ is a feasible solution to the primal and $\left(\frac{1}{3}, 3, \frac{7}{3}\right)$ is a feasible solution to the dual.

iii) Use duality to check whether $(2, 2, 2)$ is an optimal solution to the primal.